Riv. Mat. Univ. Parma, Vol. **14** (2023), 369-384

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# Sustainability in mathematical classes: A single case study on a teacher

**Abstract.** A teacher, who represents the data in this single case study, designed and implemented a learning game for a mathematics class at university. The game aimed at raising awareness about environmental and social issues concerning the cultivation of cotton, through the simulation of a real situation, lived by farmers in Texas and documented in the book by Maxine Bedat, "Unraveled: The Life and Death of a Garment". Being embedded in a mathematics lesson and, at the same time, aiming at provoking reflection about sustainability, makes the activity interdisciplinary as involving scientific, social and economical knowledge. This provoked tensions for the teacher, tensions that are analysed in this paper in terms of possible difficulties teachers can live when implementing activities of this sort, as well as in terms of opportunities for teachers' professional development.

**Keywords.** Environmental and social sustainability, mathematics education, learning mathematical modelling, mathematics teacher professional development.

**Mathematics Subject Classification:** 97M10, 97C70.

## 1 - Introduction

Climate change, population growth, pollution, resource scarcity and wastefulness are among the issues that pertain to environmental, social and economical sustainability [**8**]. Barwell [**5**] claims that mathematics plays a central

Received: June 5, 2023; accepted in revised form: January 31, 2024.

This research is part of a larger project involving researchers in mathematics education and researchers in environmental education, coordinated by prof. Chiara Andrà at the University of Eastern Piedmont, Italy.

role in dealing with these issues, as well as in responding to them. However, the reality of our mathematics classes seems to portray a dramatically different picture: Coles and colleagues [**9**] have recently pointed out that, even if environmental disasters take place frequently around us, mathematics at school seems to be taught as if nothing is happening outside the classroom walls. Grima and colleagues [**10**] point to the need for instruction that raises awareness and deep understanding of both (i) environmental, social and economical issues and (ii) mathematical competencies necessary to estimate, model and predict the phenomena related to them. Scientific, social, economical and mathematical knowledge need to be developed within an interdisciplinary approach [**10**]. Such an interdisciplinary approach unfolds issues that pertain not only to the practice of teaching mathematics within lessons aimed at fostering knowledge and also awareness, sense of responsibility and effective actions, as these are four fundamental dimensions in Environmental Education [**10**]. There are issues that pertain also to the possible theoretical approaches to teaching and learning mathematics in such contexts, as one can question the appropriateness of a theoretical framework, necessarily developed within mathematics education research, but meant to be applied to a wider theme that embraces many disciplines, such as: Physics of Climate, Ethics, Social Integration, Energy, Pollution, Chemistry. Even if mathematics, its teaching and its learning, represents the core of the didactical activities a mathematics teacher has in mind, the subject can be not at the core of the learning process the students are engaged with, and we stress that this is necessary for the activities to adhere to the needs of the classes to understand the reality.

This paper does not address such theoretical issues, even if it acknowledges that recent studies in mathematics education research highlight that, with an approach of this sort to the complexities of sustainability, it becomes a necessity to limit the so-called formatting power of mathematics [**13**]. Skovmose [**13**] argues in fact that mathematics has a strong interpretative power, in that scientific development in general can change not only interpretations of reality, but reality itself. Under this light, mathematics has a formatting power because any activity is subjugated to it, namely to the interpretation that mathematical models allow for the reality. If, on the contrary, mathematics is among the disciplines that allow for a complex understanding of sustainability issues, events become visible for their activity, preserving particular forms of knowing and doing. We conjecture that this might generate tensions for the mathematics teacher, struggling between maintaining mathematics at the forefront and dealing with sustainability issues that might leave mathematics relatively aside. In the case study under examination in this paper, a teacher designed and implemented a game-based lesson, and the tensions lived by him are the focus of our analysis. Thus, our theoretical framework is based on a review of games in mathematics classes and on teacher tensions.

## 2 - Theoretical framework

Learning games are meant for teaching skills, knowledge and attitudes rather than barely entertaining: in their review of research on learning games, Pan and colleagues [**12**] report that puzzles are the most used game genre, followed by strategy, adventure, role-play and action games. Interestingly, the authors note that the huge majority of games are employed for involving students' low-order cognitive skills (e.g., basic arithmetic computations). One exception is, in Pan's and colleagues' [**12**] review, the simulation game genre, which is a way to simulate a complex situation and is found to stimulate high-order skills such as argumentation or modelling [**12**]. Furthermore, the aforementioned review notes that the individual play mode is more frequent than the collaboration one. Therefore, games in mathematics classes are generally not used to provoke deep learning and critical thinking through collaborative activities, but they are mostly individual and for rote learning.

We conjectured that an interdisciplinary approach to mathematics learning, where the subject itself plays a relatively marginal role, generates tensions for the mathematics teacher: for example, a previous study [**3**] found that a mathematics teacher might be concerned about the lack of visible mathematics in her lesson, or the students may complain that the lesson is not that much connected to the final mathematical tests, or they may question how they would be marked after such activities. When living a tension, a teacher strives to achieve two goals that are both important, namely one cannot exclude the other, but opposed and competing [**4**]. An example of tension tied to the issues at stake in this paper is the one between the importance for the students to be prepared for the final exams and the relevance of sustainability-related issues, which require an interdisciplinary approach, where mathematics plays a background role. Tensions in teaching have been characterised as opposing forces: tensions are "expressed in terms of binaries in order to capture the sense of conflicting purpose and ambiguity held within each" [**7**] (p.120). Berlak and Berlak [**6**] offer a series of tensions intended to be useful "to citizens, researchers, parents and professionals for clarifying differences over schooling practices, and for engaging in collaborative inquiries into the origins and consequences of present patterns of schooling and the possibilities and desirability of change" [**6**] (p. 3). Building on Berlak and Berlak's [**6**] early work on tensions, others have come to see tensions as something that must be managed rather than eliminated [**1**] [**7**]. This may seem like a fine point, but the research is unanimous in its view that managing is not akin to solving, rather it is a matter of compromise and ongoing management of tensions. Ball [**4**], e.g., says that "embracing rather than trying to resolve pedagogical dilemmas gives teachers a power to shape the course and outcomes of their work with students" (p. 394). Lampert [**11**] agrees, offering her view of a teacher as a "dilemma manager who accepts conflict as endemic and even useful to her work rather than seeing it as a burden that needs to be eliminated" (p. 192).

Andrà and colleagues [**3**] find this distinction between solving and managing particularly interesting. Solving connotes a sense that there is a definite answer that would be applicable across all teaching practices. Managing suggests almost the opposite: there is no satisfactory solution, even within a single teacher's practice. However, this is not always true. Some tensions are managed, but teachers also work to resolve their tensions. Andrà and colleagues [**3**] prefer the use of the term resolve rather than solve as the latter results in a definitive correct answer, while resolving suggests a conclusion garnered from several choices, which may not please everyone but does satisfy the problem. This leaves open the possibility that some tensions are resolved and others are lived with. Andrà and colleagues [**3**] further add that tensions can be avoided in some cases: for example, a teacher may ignore the tension, or she may postpone a decision.

In Andrà's and colleagues' [**3**] study, these are general tensions but they can also hold for the specific case of game-based mathematics lessons and the teacher could search for a compromise to manage them. For example, proposing a game that simulates resource scarcity, the students might be puzzled by the novelty of the lesson and they might be worried about how the teacher would evaluate the outcomes with the activity. Or, the students might complain that such an activity does not serve the purpose of preparing for the final exams and have the feeling of loosing their time, thus creating tensions between adhering to the standards of the curriculum and carrying on a lesson that is meant for the students' understanding of the world. In the aforementioned study, interviews about the design of the lesson and about the teaching experience allowed the authors to identify and analyse the tensions of the teachers involved, and thus to contribute to the understanding of teacher's professional development more in depth. The research presented in this paper adopts a similar methodology, as it is explained in the next session.

## 3 - Methodology

The research questions we attempt to answer are: (i) which tensions emerge in the design and implementation of a game-based interdisciplinary lesson? (ii) how does the teacher manage them?

A lecturer, who designed and implemented a learning game for a class of first-year university students enrolled in an Environmental Sciences undergraduate course, is the participant of this study. His fictitious name is Federico. When he was young, he worked as a children's entertainer and, being now a teacher, he would like to exploit his experience with games in planning a lesson aimed at raising awareness about sustainability issues. This lesson is part of his first year of teaching experience.

The game is illustrated through the use of a map divided into squares (Figure 1), each of which represents a land area (hectares) that student teams can purchase and cultivate. Within the map, there are also environmental elements that can positively or negatively influence the game based on the proximity of their lands to these elements. Among these elements, there is the river (blue squares), providing water resources, but with the risk of flooding; the forest (green), offering cultivable land at a low cost, but with the risk of damaging local biodiversity; the highway (grey), near which it is not possible to practice organic farming due to car emissions, but transportation is easier; and the beach (yellow), having no productive value. These elements aim to raise awareness among students that their actions impact the territory, prompting them to make decisions that protect not only themselves and their company but also the surrounding environment. Federico believes that the map is of fundamental importance as it allows students to have a spatial understanding of the expansion of their company. In the map, hectares owned and purchased by other teams are coloured, each marked with different colours than those mentioned earlier.

The case of Federico represents for us a prototypical case study for in-service teacher education, being him at his first experiences with teaching, even if at undergraduate level. Federico was free to chose the topic, the kind of the activity, the timing and the mathematical content of the game, since his course belongs to an undergraduate program that is really innovative [**2**]. During the summer break of 2022, Federico read the book by Maxine Bedat, "Unraveled: The Life and Death of a Garment", as many students in the previous academic year raised questions about the production of textiles, both at environmental and at social level. The book shows a lot of data regarding: the cultivation of cotton, the process of transformation, the workers and it represents a rich source of data, from the past to the present, for a teacher that aims at designing a teaching activity that allows the students to better understand the process of production and distribution of a garment. In particular, in the book it is explored why, even if the organic cotton is more advantageous under many perspectives, conventional mode of cultivation is still the massive way to produce

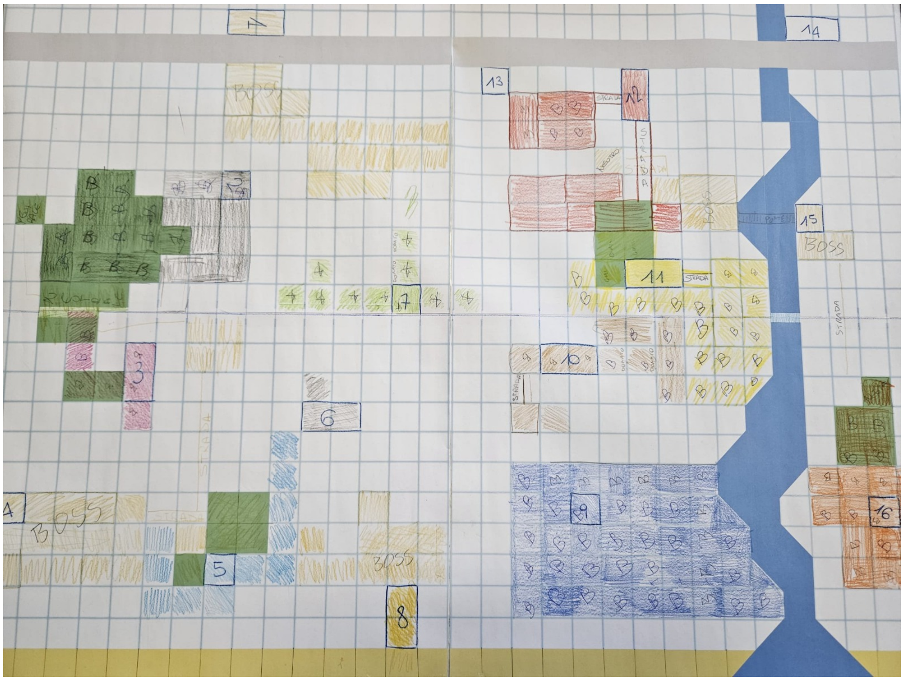


Fig. 1. An example of the map used in the game.

cotton. This is the question that the game Federico invented aims at answering.

The author of this paper conducted two semi-structured interviews with Federico (one (i) before and one (ii) after the lesson), and the questions were aimed at letting the tensions emerge from the phases of (i) lesson plan (i.e., the design of the game), (ii) lesson implementation and teacher's reflections after the lesson, respectively. The questions posed did not address explicitly the tensions, but in the first interview Federico was asked about the "big ideas" of the games, the hypothesis he had about the mathematical functioning of the game and his expectations about the strategies that the students would have used, or the difficulties they could have faced. For example, when Federico was invited to describe the choices he made in the design of the game, a tension emerged and we asked him more details about why he lives it and about how he plan to manage it in class. In the second interview, after the lesson, we asked Federico if the lesson diverted from the plan, how much and with respect to what. Since other tensions emerged, we dwelled on them and on how he managed them. In analysing the verbatim transcribed interviews, we used a qualitative coding method (see [**3**] and references therein, for more details about this method), to identify the couples of forces that constitute different tensions, and to interpret whether Federico either resolved, or managed (and managing can turn into either living with or avoiding) each tension. As we will see in the data, a tension that Federico encounters since the planning of the lesson concerns the difficulty of the mathematics to be involved in the game. One force is represented by his will to have a quick game, while the other force is represented by his will to teach mathematics that is not basic. Federico resolves this tension in favour of the first force, discarding the second one. However, during the lesson, the students themselves raise this tension and asks for more complexity in the mathematics of the game. Federico manages it by avoiding the tension, in this second case.

Before analysing the data from the interviews with Federico, we describe the lesson based on the game. The learning goals of the mathematical lesson were to recap the use of straight lines in the Cartesian plane, to reflect on the meaning of the slope, and to compare segments with different slopes. This serves as the mathematical model to analyse the trends in the budget that each fictitious farm has at the end of every year, given that the students were asked to draw the graph of each year's balance. Figure 2 shows the graphs of three farms that, in Federico's lesson plan, adopt three different strategies.

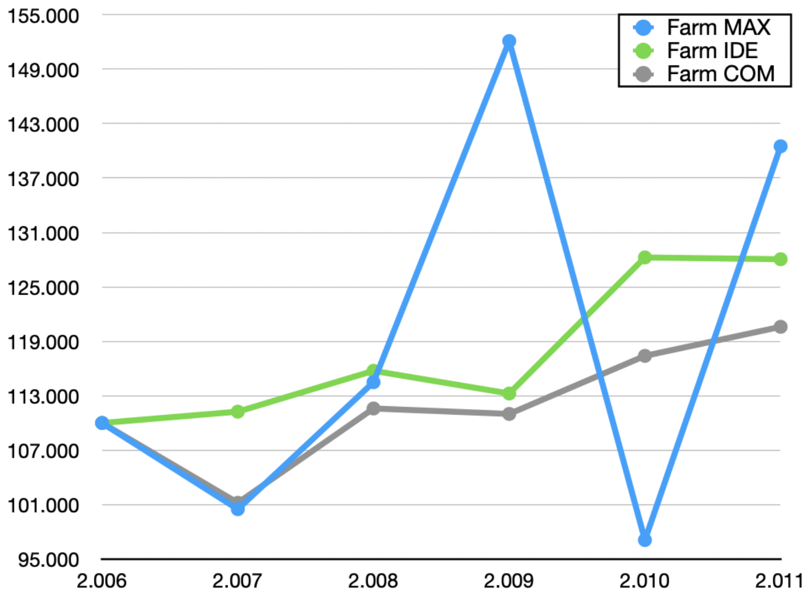


Fig. 2. The graph of three different farms, expected by Federico. The farm MAX adopts a strategy that maximises the profit and has conventional cultivation mode and not-declared workers. The farm IDE adopts an idealist strategy with organic cotton and regular, well paid workers. The farm COM searches for a compromise.

Federico imagines three possible main strategies and plans to invite the students in observing and commenting the slopes, as Federico does in the invented example shown in Figure 2, in the lesson plan. For example, the greatest values of slope (both positive and negative) emerge from farm MAX, whose strategy is based on conventional cultivation mode and undeclared workers to have a quick return even if it is higher in risk taking. Greatest values of slope for farm MAX are due to the fact that the conventional cultivation mode gives the highest profits, as well as not-declared workers, but undeclared work can be sanctioned. To note, we add that the game in this research has dual nature and function: it is both a finite product and as such it is accounted of in the methodology; and it informs the data, as Federico comments about its design and implementation, thus it is accounted for in the results and discussion sections of this paper. The lesson plan itself consists of: (i) a short phase in which slides are projected and serves the purpose to raise the question of why, even if organic cotton is better for workers, for the environment and for the balance of a farm since it gives more profits, the huge majority of cotton today is cultivated in the conventional way; (ii) the game, expected to last one hour; (iii) class discussion on both the game and the mathematics underneath. Having given the essential details about the mathematical content of the lesson and its overarching plan, we now describe the game.

As just said, the lesson starts with a short introduction about the case of Carl Pepper, a farmer in Texas who changed his cotton cultivation mode from the conventional to the organic one, described in the book by Maxine Bedat, "Unraveled: The Life and Death of a Garment". It includes some historical notions that contribute to raising both scientific knowledge about the process of production of cotton and social awareness about working conditions, as well as environmental awareness about the effects of chemical products. We recall that these are, in [**10**]'s view, central features of environmental education. Then, after some slides, the actual game starts. The rules of the game are projected by Federico in a slide and they are:

- the students are divided in groups and each group forms a farm that cultivates cotton
- each round represents one year at your farm
- at every round, unpredictable events occur and they can affect one cultivation mode or one type of workers
- for every round, each farm has to compute the balance, which is affected by the events that will be told

- the game ends in the year 2020 and the team, which will have got the most, will win
- the first year (round) is 2006 and at this time every farm has 5 fields with conventional cotton and 10 declared employees
- each field returns 1000 dollars if it is cultivated in a conventional way
- it is possible to change the cultivation mode to the organic one, but
  - one has to pay 1500 dollars for administrative cost on the first year
  - for the first three years, each field returns 500 dollars
  - from the fourth year on, each field returns 800 dollars
- it is possible to buy new fields, and their price varies
- it is possible to recruit new employees and
  - a declared employees costs 1000 dollars in the year a farm recruits her, and returns 500 dollars every year
  - a seasonal employees costs 1200 dollars in the year a farm recruits her, and returns 600 dollars every year
  - a non-declared employees costs 400 dollars in the year a farm recruits her, and returns 800 dollars every year.

The students are divided in groups of four, each one representing a farm in the year 2006 (the one before Carl changed cultivation mode). Each group has the goal of growing the farm they represent. Each farm starts with the same budget, with a conventional cultivation mode and with ten regular employees. At every round, which represents one year, the only two choices possible for each farm concern the cultivation mode (either conventional or organic) and the type of contract for the employees (either regular or undeclared). To note, the game is designed so as there does not exist a strategy that is stronger than the others.

A farm cannot convert only a portion of its field into organic mode, namely if they have 5 fields, then all the 5 fields should have the same cultivation mode. On the contrary, a farm can have some regular, declared employees and some seasonal and/or not-declared ones. Following the example of Carl, the students in the game perform actions that address environmental (the former) and social (the latter) aspects of cotton cultivation: effective actions is, indeed, another central dimension for environmental education [**10**]. Following [**12**], with respect to the play mode, we can classify the game as individual, because the choices made by the individual team do not have effects on the other teams, hence it is not collaborative. In fact, within each team the students collaborate and reach a shared strategy, but between the teams there is no interaction and we interpret this feature of the game as individual play mode, in the sense that the choices made by an individual team do not have any effect on the choices made by the other teams.

At every round of the game, each farm computes the revenue of the corresponding year. The revenue is a linear function of the number and type of employees and the cultivation mode in a year. If the farm changes its strategy or unpredicted events occur, the slope of the segment changes. The teacher plays the role of the market. Every year unexpected events occur (e.g., drought), and constitute an advantage/disadvantage for some teams: for example, some events represent an advantage for those farms who have a conventional cultivation mode, while do not affect the organic ones, or represent and advantage. Other events affect only undeclared type of contract for their employees. If we take conventional cultivation as an example, it is advantaged in terms of the speed of growth of the cotton plants and recovery in case of unforeseen natural events (as it can be seen also in Figure 2), but it over exploits the soil and risks to harm its employees (diseases caused by chemicals is another example of unexpected events). A company that adopts an organic cultivation faces economic challenges for the maintenance of its production, risking failure and therefore not completing the game, but it is advantaged by government subsidies (a third example of unexpected events), which in turn promote growth and development. The teachers gives notice of possible unexpected events at the beginning of every year. We stress that this game belongs mainly to the simulation game genre, and we add that the mathematics embedded in the game is intrinsic [**12**], as it tends to remain invisible, while sustainability issues related to the context of the game play a prominent role.

## 4 - Results and analysis

We report and analyse the parts, in the two interviews with Federico, where tensions emerge. As said in the Methodology, the tensions were identified by looking at possible couples of opposing forces that Federico explicitly mentioned. Each interview lasted about 30 minutes and, all in all, we report about 90% of the ideas that Federico shared. In the interview about the lesson plan, Federico spends a considerable amount of time arguing about his choice of the kind of mathematics involved in the game, as he knows that arithmetical concepts are relatively easy for his students, and running a real farm may entail more complex mathematics (i.e., this is a tension), but he opted for tasks in the game where, he says, "mathematics plays a purely computational and monetary management role to increase the production of the enterprise". Low-order computational skills are involved, instead of high-order modelling and reasoning ones (to note, low- and high-order skills are understood in terms of [**12**]). Federico adds: "by developing and growing their company round after round, the teams find themselves calculating their budget, expenses, and revenue, resorting to sums and multiplications". To recall, the mathematical learning goal of the activity is to recap the use of segments in the Cartesian plane and the meaning of the slope, hence no new mathematical topic is introduced. We comment that mathematical knowledge, among the dimensions relevant for environmental education [**10**], emerges as a tool to play the game (without sums and multiplications a team does not pass the round). Moreover, in Federico's words, "the few simple calculations allow a quick play without interruptions". This, from his standpoint, "helps to maintain a greater concentration on the choices to be made (not only economic, but also ethical) and greater communication between the mates of the same team who, talking to each other, develop the company making the choices they prefer". However, Federico is struggling with having more high-order cognitive skills involved, especially because the story of Carl Pepper represents an opportunity to provoke reflection and discussion and "the risk is that it remains unexplored", he declares. We can notice that a tension is emerging: between high-order cognitive skills triggered by the complexity of the real world, and low-order ones embedded in the basic mathematical computations. Federico seems, however, to resolve the tension and chose one force instead of the other.

In the second interview, at the end of the lesson, Federico notices: "the students were immersed and involved in the activities, everyone was participating". However, almost at the end of the lesson, the tension between low- and high-order cognitive skills surfaces again and this time the students raise it. In fact, they complain about the ease of the computations to be made. As it emerges in the lesson plan, Federico believes that low-order cognitive tasks allow the game to be quick, while students' expectations are more in favour of high-order ones. Federico also comments that: "if the game is not challenging enough, the students can lose their motivation and their engagement in it". He also adds that, next time, he would allow for more complex choices to be made by the teams (i.e., he would not only leave open the possibility of choosing between organic and conventional cultivation, and between regular and undeclared contracts, but to have more options available for the teams). To note, Federico is not increasing the complexity of mathematics in the game, but rather the complexity of the choices, thus we can comment that he is still resolving the tension between low- and high-order mathematical skills in favour of the former force, but at the same time he is struggling to resolve the tension between low-order mathematical tasks and high-order ones involving reflection and connections to the real world.

In his interview, we can identify three goals that Federico has: to keep the students engaged; to include meaningful mathematics; and to complete the game quickly. While the first two do not appear to conflict, they both are in conflict with the third and this gives raise to tension. Thus, we have two tensions and we firstly dwell on the one between students' engagement and quick game. It is interesting to note that, during the lesson, the students invented ways to solve issues in the game, ways that have not been foreseen by Federico and were not part of the rules. An example is given by two teams deciding to help a third one to prevent bankruptcy (this kind of actions was not part of the game as it was designed). These actions were carried out by the students without any prompt from the professor who, despite being the moderator of the game, did not intervene to change the rules, nor to punish these choices, because the rules of the game were few and limited to regulate the types of cultivation and contract. Why did Federico acted in this way? In the interview after the lesson, Federico further commented: "Today many companies have workers on different contracts. The fact of implementing them in the game would make it more varied and truthful" and adds that he allowed the students to invent new rules because he wanted to explore with them the potentialities of this. Federico also comments that greater customisation of the company would make the game more engaging, realistic and more in line with today's business models: "each company would have its own development and at the end of the game we would reach a variety of situations". In these worlds, we see a confirmation that Federico is prone to resolve the tension in favour of high-order non-mathematical cognitive skills.

We now comment on the tension between low-order mathematical skills in a quick game, and high-order mathematical skills in a less quick game, which we argue that is avoided by Federico. To have "more mathematics" is part of the dilemmas Federico makes explicit in the lesson plan. Also in the second interview, he adds: "the question I still ask myself is whether, by increasing the difficulty of mathematics and asking to draw graphs, there is a risk of making the activity boring and unattractive. The risk is also that students lose focus on the ethical aspects of their choices, which is my goal." From these words, we can see that Federico is avoiding the tension, because he "asks himself" whether to insert more mathematical tasks, but he seems to be willing to postpone a definite choice, as if he is trying to ignore the students' complaints about the "too simple" mathematics involved.

A third tension emerges, during the lesson, from the difficulty of managing the numerous teams (n=22) in the classroom: "changes in the type of cultivation, contracts to be signed with new employees, answering to the students' questions, listening to the students' request for the possibility of selling or buying parcels of land: these are factors that have prevented the linear course of the game, forcing me to interrupt frequently to re-explain the rules or to modify them when I noticed that the students risked getting bored and not completing the game".

During the lesson, Federico was trapped in the tension between clarity and inclusion of every student (having had to stop frequently to explain or to change the rules so that everybody was engaged), and the rhythm of the game (in the interview before the lesson, it emerges that Federico wanted a quick game). In his words after the lesson, it seems that Federico would live with this tension by managing it in this way: "For the future, I want to implement tools to help the teams running their business tasks better, making them more autonomous in making their choices, and avoiding wasting the professor's time by asking questions, while he has to focus on the rhythm of the game. A simple but effective strategy could be to give them a sheet with a summary of the rules, the name of their company (also useful for recognising the teams) and a section used to write down their earnings for each round based on their purchased cotton and hired employees".

We interpret this as a case of living with tension, because the one between following every team's work and maintaining an overarching view of the game is unresolvable per se, but the strategies mentioned by Federico can support him living with it.

## 5 - Discussion and conclusions

In this paper, we analyse the tensions that emerge from the design and the implementation of a game-based lesson on sustainability issues. The validity of this kind of activities is justified by the framework, but the paper draws attention to the tensions of the teacher who runs the game: he need a broad knowledge of the topics covered, he needs to ensure that the teams' calculations are correct, to administer any modifications, and to maintain the students' concentration and engagement high. The results, thus, help us to better understand possible obstacles that teachers can face when teaching an interdisciplinary lesson on sustainability issues, and thus that can prevent teachers enacting such a teaching. The main result of this research is, thus, pairs of opposing forces identified when a sustainability-related mathematics lesson based on a simulation game [**12**] is designed and implemented. The four dimensions of students' managing sustainability issues [**10**] are also used to code Federico's comments. The first two tensions we analysed pivot around the complexity of the real world and the difficulty of the mathematics embedded in the game. The tension is partly anticipated also in the first interview with Federico and, following [**6**], we can say that its emergence in the lesson plan helps us clarify the origins of certain pedagogical choices instead of others. Reflecting after the lesson, Federico attempts a compromise that takes into account the students' desires, for the first tension, whilst for the second one Federico tends to avoid it. With [**3**] we can say that these are two cases of tensions that strive to be resolved, rather than just lived by. The students, in fact, push for making the most of their knowledge (in [**10**]'s terms) in the game, in both cases. Possible implications for teaching are, thus, that in designing a game-based lesson a teacher should introduce some complexity both in the mathematics and in the real situation the game refers to.

We recall that the students raise two tensions for the teacher, both pushing for higher-order cognitive challenge: at the level of the range of possible real-life choices that can be made in the first case, at the mathematical level in the second one. The first tension involves not only knowledge, but also actions, sense of responsibility and awareness [**10**], and Federico tries to resolve it in favour of the students' requests: with [**11**] we can say that the emergence of a tension is useful to the teacher's work as it promotes change in favour of complexity. We also argue that, however, there is a significant difference between the two tensions: for the second, mathematical one, Federico seems more reluctant to really change and add "some more mathematics", while for the first one, socially centred, he proposes concrete changes right after the lesson. Federico could have been prompted in doing this by the solutions that the students invented in the lesson, with respect to having more social complexity. Students' actions can represent a strong stimulus to enact change. Thence, another implication of this study can be that a fixed, pre-packaged game brings with it greater tensions from the point of view of the professor, whose interest is inclusiveness and clarity for all, than completely adapting to his students' demands.

The third tension is between clarity/inclusion and pace of the game. Federico explains how he would manage it and this is an example of higher awareness in the actual possibilities for class management embodied in the intention to prepare sheets of paper with the rules and a table to report the computation in a structured manner), prompted by the emergence of the tension. We comment that this tension can be related to Federico's lack of teaching experience.

Despite the various tensions that emerged, Federico notices that the students were involved and, with [**4**], we can comment that such tensions result in powerful tools, in the hands of the teacher, to shape the course and the outcomes of the work with the students. Living by the tensions emerging from the lesson, Federico manages the activity to be welcomed by the classroom: the students took an active part in the game, developing original solutions that have not been anticipated.

We would like to stress that the game allows us to focus on the tensions that a teacher can leave when planning a lesson of this sort, a lesson that takes [**9**]'s challenge of not ignoring, but bringing in a mathematics class, environmental and social issues that are at stake in today's world.

*Acknowledgments.* The author thanks the teacher, whose nickname is Federico, for his availability to be interviewed and for all the teaching material he shared with the author, making this publication possible. The author also thanks Giulia Lambertini, who shared a first version of the game and who, with Chiara Andrà, supervised the design of the activity and of the interview. Finally, the author thanks Chiara Andrà, Luca Doria and Andrea Amico, for their useful comments in the process of writing of this paper.

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