Riv. Mat. Univ. Parma, Vol. **14** (2023), 355-368

Chiara Andrà and Francesca Martignone

Challenges in modelling in early years of school: The case of Adele

Abstract. The present paper aims at discussing the challenges for teachers in teaching mathematical modelling in primary school. We first argue about a working definition of mathematical modeling in teaching-learing activites carried out in the early years of school. Subsequently, a model for the specialised knowledge of mathematics teacher is recalled and specified to the case of modelling in primary school, and a single case of a teacher, fictitiously named Adele, is analysed as an example. This case study allows us to link and highlight some aspects related to a teacher's knowledge and beliefs about teaching and learning mathematics and in particular about modelling activities in primary school.

Keywords. Mathematics education, primary school teaching, learning mathematical modelling.

Mathematics Subject Classification: 97M10, 97C70.

1 - Introduction and theoretical background

As Blum and Niss [**4**] noted some years ago, mathematics education researchers have regularly advocated the inclusion of applications, modelling and problem solving in mathematics instruction, and sometimes this has been realised in some curricula. The reasons for embedding mathematical modelling in school mathematics curricula include: (i) preparing students to live in a society the functioning of which is being increasingly influenced by the utilisation of mathematics through applications and modelling; (ii) the non-automatic activation and application of pure mathematical knowledge to extra-mathematical

Received: April 28, 2022; accepted in revised form: April 28, 2023.
This research was partially supported by Fondi di Ateneo per la Ricerca (FAR2017), University of Eastern Piedmont.

situation; and (iii) the importance to establish with students a rich and comprehensive picture of mathematics in all its facets, as a science, as a field of activity in society and culture. More recently, [**11**] argued that the competencies to solve real-world problems using mathematics (i.e., modelling competencies) are often quoted in the goals for the development of teaching-learning activities in many national curricula. “However, beyond this consensus on the relevance of modelling, it is still disputed how to integrate mathematical modelling into the teaching and learning processes” [**11**] (p.20). These considerations become even more relevant if we focus on primary school, since we need a definition of modelling activities that takes into consideration the specific potentialities and limits of teaching and learning mathematics in grades 1-5. Thence, we now recall definitions of mathematical modelling developed within mathematics education, and then reframe them with respect to primary school mathematics.

The core of modelling activities in mathematics can be described as transfer processes between reality and mathematics [**3**] [**13**]. One of the most interesting models proposed, within Mathematics Education, to describe modelling activities is the cycle elaborated by Blum and colleagues [**3**], which consists of a seven-step sequence of activities that are: (1) understanding the problem and constructing an individual situation model; (2) simplifying and structuring the situation model and thus constructing a real model; (3) mathematising, i.e. translating the real model into a mathematical model; (4) applying mathematical procedures in order to derive a result; (5) interpreting this mathematical result with regard to reality and thus attaining a real result; (6) validating this result with reference to the original situation and, in case the result is unsatisfactory, the process can start again with step 2; (7) exposing the whole solution process. [**15**] notice that identifying and distilling these seven steps is helpful for reconstructing the modelling processes used by students when solving mathematical problems. However, as [**5**] note, the students, in actual modelling processes, typically do not follow a linear path, but rather jump back and forth several times between mathematics and reality. However, an important advantage of the seven-step modelling cycle is argued by [**15**] to be the separation between constructing a situation model, a real model, and a mathematical model. This distinction, in Schukajlow et al.’s view, allows for identifying various sources of difficulty for students: i.e., difficulties in understanding the given situation, in simplifying and structuring the information extracted from the situation, and in choosing a suitable mathematical description of the situation during students’ solution processes. As a consequence, this model helps teachers in choosing appropriate, well-aimed and adaptive interventions especially in the critical translation phase at the beginning of the modelling process.

The focus of the research presented in this paper is on primary teachers' teaching of modelling in mathematics. If we stick to the Italian context, we can notice that, in the goals for the development of competencies in the Italian National Guidelines [**9**], there are no explicit statements about the activity of mathematical modelling, but in the goals we find features that recall aspects of modelling. In fact, we can read: "[A student] searches data to obtain information and to constructs representations (tables and graphs). She can also draw information from data represented in tables and graphs" and "she can solve easy problems in all content areas, keeping control over both the process of solving them and the results" (translation by the authors) (p. 49). Drawing on these premises, for grades 1-5, we are prompted to consider an alternative, yet more modest, definition of modelling, which is called by [**15**] "dressed up" word problems. In this kind of problems, "the reality-related mental activities are much simpler than in modelling problems since the simplified real model is already given from the beginning by the description of the problem" (p.219), and the validation of the result with respect to the real situation is also easier. When dealing with this problems, students do not need to make assumptions about missing data or about selecting relevant data, and the process of validation of the results is mostly limited to checking the mathematical part. However, we add (and contrast) to Schukajlow et al.'s [**15**] considerations that mathematical modelling can take place, in elementary mathematics education, in situations in which the real world plays a foreground role and the mathematics takes a marginal role, given the embodied and factual layer of mathematics generality that is possible to reach in primary school [**14**].

In this paper, we thus aim at elaborating on the kind of mathematical modeling that can be actually done in primary schools, and we do so by analysing a single case study: the case of Adele. Adele's reflections on the modelling activities she carried out in a grade-5 class allowed us to investigate and highlight different aspects of her specialized knowledge and beliefs about mathematics teaching and learning. To do this, we will use Carillo et al.'s [**7**] model for the mathematics teachers' specialised knowledge, which we apply to mathematical modelling. We now recall it.

2 - The theoretical framework of mathematics teacher's specialised knowledge

One of the key findings of Carillo et al.'s [**7**] studies is the possibility, offered by their model and endorsed by the teachers participating to their research, to delve more deeply into the knowledge usable for teaching. The starting point of their research is that, for a teacher to carry out her role (namely, planning the lessons, collaborating with colleagues, giving lessons and taking time to reflect on them afterwards), it emerges a need for specialised knowledge for and in teaching mathematics.

Inspired by [**16**], the Mathematics Teacher's Specialised Knowledge (MTSK) model [**7**] considers two main areas of knowledge: the Mathematical Knowledge (MK) and the Pedagogical Content Knowledge (PCK).

[**7**] argue that teachers' actions are strongly related not only to what they know about mathematics, but also to their conceptions and beliefs [**18**] about mathematics, how it is learnt and how it should be taught. A more or less coherent set of beliefs permeates a teacher's knowledge in each of the aforementioned sub-domains. Therefore, the MTSK model also includes beliefs about mathematics and about mathematics teaching and learning, and beliefs and knowledge are understood as reciprocal to each other.
To this respect, with a focus on teaching mathematical modeling at primary school, a critical aspect can emerge, that is: being a schema-oriented view [**8**] dominant among primary school mathematics teachers [**1**], we can see a strong resistance in teachers' attitudes and beliefs against applied mathematics. However, as [**6**] note, positive evidence from a teacher's practice can drastically change her beliefs: for example, noticing a positive effect on the students' motivation and engagement in a mathematical activity concerning modeling can drive the teacher to systematically introduce modelling in her teaching practice. This represents for us a main focus in our research.

Having recalled the main basic features of the MTSK model, we now delve on each subdomain (borrowing from Carrillo's and his colleagues' work) and attempt to characterise each one with respect to mathematical modelling (offering our insights).

2.1 - *The MK domain and its subdomains*

In the MTSK model [**7**], mathematics is defined as "a network of systemic knowledge structured according to its own rules" (p.6). According to this definition, the mathematical domain (MK) can be split into three subdomains, which are: (i) the mathematics content itself, namely the knowledge of the topic, (ii) the knowledge of the structure of mathematics, and (iii) the knowledge of the practices in mathematics.

With reference to mathematical modelling, we argue that the first subdomain includes: the type of mathematical contents that can be used to model particular contexts and situations, as well as possible ways that can be used to represent these contents, and possible connections to elements of the same topic that are marginal to solve a particular problem, or to model a particular situation, but are important to understand the mathematical content in its entirety. An example, borrowed from [**7**] is "the teacher's knowledge of different contexts associated with the concept of fraction and its meanings. Likewise, included within the teacher's phenomenological knowledge of the topic would be their awareness of its uses and applications" (p. 242). Another interesting element within the knowledge of topics, in the MTSK model, is the knowledge of the procedures involved in a topic. "This includes knowledge of how to do something (e.g. algorithms, both conventional and alternative), when to do something (the sufficient and necessary conditions to apply an algorithm), why something is done (the principles underlying algorithms), and the characteristics of the resulting object" (p. 242). We claim that this definition fits well within Schukajlow's and colleagues' [**15**] understanding of mathematical modelling. We also claim that, for a primary school mathematics teacher, a source of difficulty can arise within this sub-domain of MK with respect to specific mathematical modelling as, from one side, primary school students may resort to a limited "set" of mathematical topics and procedures, being their mathematical knowledge limited, and from the other side also a primary mathematics teacher can have limited mathematical knowledge [**2**] to use in designing and carrying out lessons dedicated to mathematical modelling. Hence, in order for a primary school teacher to use her knowledge to teach mathematical modelling requires dedicated effort, which needs to be acknowledged.

2.2 - *Pedagogical content knowledge (PCK)*

The MTSK model [**7**] considers PCK as the knowledge in which the mathematical content determines the teaching and learning and specifies three subdomains: Knowledge of Mathematics Teaching (KMT), Knowledge of Features of Learning Mathematics (KFLM), and Knowledge of Mathematics Learning Standards (KMLS). The first sub-domain encompasses knowledge associated with features inherent to learning mathematics, placing the focus on mathematical content (as the object of learning) rather than on the learner. The main sources of teachers' knowledge within this subdomain tend to be their own experience built up over time along with research results in Mathematics Education.

To this respect, another critical issue can emerge for teaching modelling activities in primary school that is: a lack of working examples and/or previous experiences to be adapted to her specific class, since modeling in Italy is often a not very extensive activity introduced in primary schools.

KFLM takes account of the teacher's knowledge about their students' manner of reasoning and proceeding in mathematics (in particular, their errors, areas of difficulty and misconceptions), which in turn informs her interpretation of their output. KFLM incorporates knowledge of learning styles and different ways of perceiving the traits inherent in certain content. Along the same lines, the sub-domain includes theories, both personal and institutionalised, about students' cognitive development with respect to both mathematics in general and specific content. The range of knowledge comprising KFLM also includes the procedures and strategies, whether conventional or unconventional, that students use to do mathematics, as well as the terminology used to talk about specific contents: in short, the different ways in which pupils interact with mathematical content.

Specific to the content, we argue that in introducing mathematical modelling for the first time in her classes, a teacher can further face difficulties with respect to KFLM, as she might not have a repertoire of typical students' errors, difficulties and misconceptions. Moreover, since teacher's beliefs can act as a barrier to change, it becomes important also the way(s) used to introduce mathematical modelling activities to teachers, for example in a PD course [**17**].

Finally, as far as KMLS is concerned, it is interesting to note that the MTSK model does not include in this sub-domain only the knowledge of national curricula but also of documents such as NCTM standards. Specifically for modelling, KMLS can comprise a teacher's knowledge of other International documents. Moreover, this holds true not only for a teacher's own school level but for all levels: this is important in the planning phases, since in the planning of educational activities one works in terms of prerequisites and objectives, keeping in mind both the previous grades (prerequisites), and the future ones (objectives). Specifically for modelling and the Italian case, even if we have seen that it is not explicitly mentioned in the National Guidelines, modelling is part of middle and high school mathematics, so a teacher can introduce modelling by looking at the goals stated for grades that are subsequent to the ones she is teaching.

3 - Methodology

The participant of this study is a primary school teacher fictitiously named Adele, who participated to a PD course led by the two authors in the school year 2020-21. She was part of a group of 8 teachers attending the PD, and we selected her case because she visibly changed her beliefs about mathematics and teaching and learning processes during the PD.

The course lasted from September 2020 to May 2021: 6 hours were dedicated to workshops (at distance, because of the pandemic situation, and coordinated by the teacher educators) and 17 hours were dedicated to teachers' project works and classroom activities. Finally, at the end of the course, in a 2-hour meeting, the teachers presented and analysed materials collected in the class activities (previously also shared via a Moodle platform). The materials shared and discussed were: photographs of student works, and presentations in which teachers described their classroom activities.

The course was designed in synergy with the school where the teachers work, most of whom were in their first year of service at that school. In view of the situation due to the pandemic, we wanted to design courses that would integrate the new technologies and yet maintain activities with concrete artifacts that students have at home (e.g., pasta). For this reason, in the first part of the course we explored and analysed tools from Google Classroom, in particular we used presentations and Jamboard.

The course was divided into two phases, a first phase dedicated to the study of technologies and a second part to the design and development of activities in the classrooms. In the first phase, therefore, one of the teachers, who was also an expert primary school teacher education, conducted learning activities linked to the use of new technologies and at the same time the two authors of the paper (who were the other two teacher educators) were responsible for involving the teachers in reflections on teaching methods to be developed in the classroom, also in flipped classroom mode. In the second part of the course the authors reflected with the teachers on the use of artefacts in laboratory activity and on the development of problem solving, stressing the importance of enhancing tasks in which there may be several solutions. All this is in line with the Italian National Guidelines. Some examples of activities in which students could count and then also estimate quantities of pasta were presented. These choices were made because teachers worked in classes of different grades (from kindergarten to grade 5) and counting activities can be developed vertically. Each teacher then independently designed the activities in their classes, following the shared objectives drawn from the national guidelines and keeping the use of pasta as a common thread. Thus, no activities proposed by the teacher educators were reproduced, but each adapted the shared suggestions and reflections to his or her own context.

After the end of the PD, we decided to interview Adele, because at the beginning of the PD she was doubtful about the activities she could do, but at the end of the course she enthusiastically presented a very rich activity. The interview was semi-structured and lasted about 60 minutes. It was video-recorded and then fully transcribed. The structure of the interview aimed at letting Adele's beliefs and experiences emerge through the narrative rather than by direct questioning. In particular, we asked Adele to recall the main aspects of the activity she conducted, and to compare it to her usual practice before the PD.

The interview with Adele was planned by the authors in this way: the authors agreed on possible hints that would allow them to frame the interview with Adele, and on some specific aspects they wanted to investigate and on which they would have asked direct questions if they had not emerged from Adele's narrative, in particular regarding her reflections on teaching-learning mathematics, on how she had experienced it over the years and on what had changed during and after the PD led the authors. The authors started by explaining to Adele the interest in researching her activity and asking her to review together the material she had collected during the activity, so that Adele had the opportunity to also share her reflections afterwards. During this review, Adele described the activities and motivated her teaching choices. We recalled together her initial reticence and Adele expressed without being asked directly what her convictions were about teaching-learning mathematics and what she considered fundamental in her relationship with her students.

We interpreted Adele's statements in the interview through a qualitative coding method [**12**], applying the categories of the MTSK model adapted to the case of mathematical modelling.

4 - The case of Adele: data presentation and data analysis

Adele taught for 20 years at Kindergarten, and the school year 2020-21 was her first one in primary school. She was teaching in grade-5. In this paragraph we will retrace the account, she gave during the interview, of her experience, analysing the different aspects that emerged according to different theoretical lenses. She said that, with Kindergarten children, she was used to teach mathematics with manipulatives of any sort: "At kindergarten, students learning originates from everyday tools, it fosters creativity and it is really physical. These things were natural for me" –said Adele in the interview conducted after the end of the PD and, resorting to the MTSK model, we can notice an emergence of the specialized knowledge she had developed with respect to mathematics teaching in Kindergarten. For teaching in grade-5, she said that her strongest belief (among her beliefs on mathematics teaching and learning) was that students had to learn mathematics through predominantly mental activities. This belief was reinforced by her awareness that her students should be ready for middle school (to note, middle school in Italy comprises grades 6-8), so she wanted to adhere to the National Guidelines the most possible with respect to the contents to be taught. "I came to the PD with some degree of worry and stress, because I had a grade-5", said Adele in the interview, confirming what we noticed in the first meeting with the teachers in the course of the

PD. Under the lens of the MTSK model, we notice the strong relation between Adele's beliefs about the kind of mathematics learning that should take place in grade-5, as well as her knowledge of the learning standards (KMLS) in Italy.

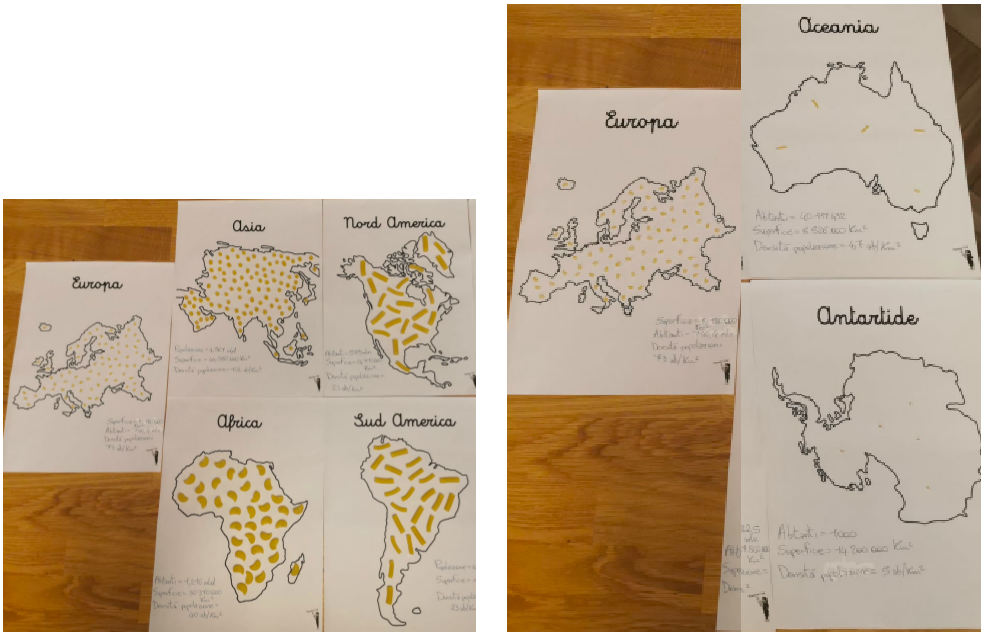
In that year, she had to teach geography to grade-5 students and "geography — one can read in the National Guidelines [**10**] — represents a "hinge" between humanistic and scientific disciplines. Many tools, words, methods and some contexts for inquiry make geography close to mathematics, sciences and technology; however, geography explains also the interaction between the man and his environment, the choices of the communities, the migrations, the flows of raw materials and of resources, and all this makes geography close to antropology and social sciences" (translation of the authors) (p. 11). In particular, Adele had to introduce the concept of density (i.e., population density). Adele was about to dropout from the PD, as, despite the hints from the National Guidelines about the connections between mathematics and geography, she found no way to develop a teaching activity pivoting around the use of pasta in mathematics. In Adele's view, the modelling activities have pedagogical goals that are strongly connected to teaching modelling, i.e. she wants to develop abilities that enable students to understand central aspects of our world in a better way [**3**] [**11**]. However, at a certain point of the PD, Adele felt far from realising such goals. The first author, playing the role of a facilitator during the PD [**17**], acknowledged Adele's difficulties, but also proposed to consider population density as a mathematical concept that could be approached and visualised through pasta, resorting to the students' creativity. "This changed my life" – told Adele in her interview. "Yeah, I completely changed my approach to teaching and I went back to the teaching methods I was used to in Kintergarden. And I figured out that you were right: also children at this age learn through their body". Under the lens of the MTSK model, we can see a strong intertwining of beliefs and knowledge of learning mathematics (KFLM), as well as a relief of a tension that Adele was living with respect to how to teach in grade-5. During the PD course, she found a confirmation of the importance of the involvement of students in concrete activities and thus Adele seems to give less importance to her initial belief about the centrality of the mind in grade-5, and to embrace, also in teaching-learning activities for primary school students, the manipulative practices she was used to. Therefore, she used her knowledge of teaching (KMT) developed during her past experiences in Kindergarten.

At the beginning of the class activity, Adele tries to convey the concept of density through somewhat abstract graphic representations, but this "was not of help with my students". According to Adele, it was the activities with the pasta that would have provided the most effective understanding of the concept of density. In the activity carried out by Adele, density is not just a number obtained through arithmetic operations. Using Vergnaud's [**19**] definition of the concept, the concept of density is explored and understood through different components that define it, namely reference situations, representations and schemes, i.e. invariant behaviour in classes of similar situations. Through the creation of concrete models, the concept of density is given meaning and students can reflect on how that numerical value can give information about a real phenomenon.

In order to better understand what Adele did, let us now look at the details of the activity carried out in Adele's class. The teacher prepared a short presentation with Power Point, which was shared in Google classroom (to note, the context of the pandemic led teachers to be always ready to teach at distance, or with some students at home). Not only Adele has good technological knowledge, but she also shares a belief with us: "I use presentations, because images are a fundamental tool for learning". Each slide of the presentation served the purpose to engage the class in a brainstorming activity: "the students and myself make to each other questions and the kids gave deep-thinking answers". In describing this part of the activity, Adele also adds: "kids love to talk and to participate".

The very core of the activity consists in the production of a map of the world with pasta representing the various densities of populations living in different continents. "This was given as an homework to the students" —said Adele, justifying her choice, made with some regret, due to the pandemic situation. Figure 1 shows two examples of homework done by Adele's students who, in the teachers' own words "sent the pictures of their work right after the end of the school, on the same afternoon the homework was assigned". From this statement, we can see that the interview with the teacher reveals that her aim was also to build a positive attitude towards mathematics, through activities linking the understanding of the real world and the use of concrete representations to explore the concept of density. Once again, this can be linked to both Adele's beliefs according to the MTSK model, but also to her knowledge of learning standards (KMLS), as to develop a positive attitude towards mathematics is explicitly stated in the National Guidelines: "*Goals for the development of competences at the end of primary school*: A student develops a positive attitude towards mathematics, through significant experiences, which have made him/her understand how the mathematical tools he/she has learned to use are useful to operate in everyday life" (p.50).

A close look to Figure 1a shows that the student embodied the concept of density to, somehow, the space available to one ideal single person. Namely, in Europe pasta is smaller since people have less space and density is higher,



(a) (b)

Fig. 1. (a) the picture to the left represents an homework from one student and (b) the picture to the right represents another work. We can notice that two different uses of the sizes of pasta to convey the notion of density have been made: in case (a) the dimension of the pasta is proportional to the "available space" for each person, namely the larger the pasta, the lower the density, while in case (b) the pasta have similar dimensions and the space between the pasta conveys the idea of density, namely the more the pasta (and the less the blanc space), the higher the density.

whilst in Africa pasta is bigger because people have more space and density is lower. In figure 1b, the concept of density is related to the blanc space between the pieces of pasta and for Oceania there are a few pieces of pasta and more blanc space because density is lower with respect to Europe, where the blanc space is less. In Adele's view, both works have been valued as being really good, as for her it is important that each child develops her own understanding of mathematics with respect to reality.

5 - Final reflections

In this paper, we discussed possible characteristics of modelling activities that can be carried out in primary school and we focused on the knowledge teachers can resort to in these grades. Our analysis deals with specialised knowledge and beliefs of a primary teacher. We used the MTSK model to highlight, in the different subdomains of mathematical knowledge and PCK, aspects of modelling in mathematics. Moreover, the MTSK model allowed us to highlight the centrality of teachers' beliefs and provided us with interpretative lens that we used in the analysis of a single case study. We focused on the teacher's knowledge and beliefs before and after an experience that also involved modelling processes with concrete objects. The beliefs stated by Adele, during an interview carried out after the classroom activities, are strongly intertwined with her content and pedagogical knowledge. Adele's interview also highlighted how her beliefs influenced her teaching choices and how a particular modelling activity modified her beliefs about teaching-learning mathematics as well as consolidating her pedagogical content knowledge.

This contribution is only a first step in the exploration of these issues, in particular future research will aim to investigate, in more depth and on more cases, possible changes in beliefs and specialised knowledge of primary school teachers and how these changes could be influenced by activities in which the use of artefacts in modelling activities is required.

Acknowledgments. The authors thanks the teacher, whose nickname is Adele, for her availability to be interviewed and for all the teaching material she shared with us, making this publication possible.

References

[**1**] C. Andrà, F. Arzarello, L. Bazzini, F. Ferrara, D. Merlo, C. Sabena, K. Salvioli and B. Villa, *Sketching primary school teachers? profiles*, In: F. Furinghetti & F. Morselli, eds, "MAVI-15: Ongoing research on mathematical beliefs", Genova, 2010, 79–88.

[**2**] C. Andrà, A. Rouleau, P. Liljedahl and P. Di Martino, *An affective lens for tensions emerging from teacher professional development*, For the learning of mathematics **39** (2019), 2–6.

[**3**] W. Blum, P. Galbraith, W. Henn and M. Niss, *Mathematical Modelling and Applications in Mathematics Education*, Springer, New York, 2007.

[**4**] W. Blum and M. Niss, *Applied mathematical problem solving, modelling, applications, and links to other subjects – State, trends and issues in mathematics instruction*, Educational Studies in Mathematics **22** (1991), 37–68.

[**5**] R. Borromeo Ferri, *Individual modelling routes of pupils ? Analysis of modelling problems in mathematical lessons from a cognitive perspective*, In: C. Haines, P. Galbraith, W. Blum & S. Khan , eds, "Mathematical Modelling

(ICTMA 12): Education, Engineering and Economics", Horwood Publishing, Chichester, 2007, 260–270.

[**6**] D. Brunetto, G. Bernardi, C. Andrà and P. Liljedahl, *Teaching as a system: COVID-19 as a lens into teacher change*, Educational Studies in Mathematics **110** (2022), 65–81.

[**7**] J. Carrillo-Yañez, N. Climent, M. Montes, L. C. Contreras, E. Flores-Medrano, D. Escudero-Ávila, D. Vasco, N. Rojas, P. Flores, Á. Aguilar-González, M. Ribeiro and M. Cinta Muñoz-Catalán, *The mathematics teacher's specialised knowledge (MTSK) model*, Research in Mathematics Education **20** (2018), 236–253.

[**8**] R. Erens and A. Eichler, *Belief changes in the transition from university studies to school practice*, In: M. Hannula, G. Leder, F. Morselli, M. Vollstedt & Q. Zhang, eds, "Affect and mathematics education", ICME-13 Monographs, Springer, Cham, 2019, 345–373.

[**9**] *Indicazioni nazionali per il curricolo della scuola dell'infanzia e del primo ciclo d'istruzione*, Ministero dell'Istruzione dell'Università e della Ricerca, Roma, Decreto Ministeriale 16 novembre 2012, n. 254.

[**10**] *Indicazioni nazionali e nuovi scenari*, Documento a cura del Comitato Scientifico Nazionale per le Indicazioni Nazionali per il curricolo della scuola dell'infanzia e del primo ciclo di istruzione, Ministero dell'Istruzione dell'Università e della Ricerca, Roma, 22 febbraio 2018.

[**11**] G. Kaiser, *Mathematical modelling and applications in education*, In: Lerman S., ed., "Encyclopedia of Mathematics Education", Springer, Dordrecht, 2014, 396–404.

[**12**] P. Mayring, *Qualitative content analysis: Theoretical background and procedures*, In: A. Bikner-Ahsbahs, C. Knipping & N. Presmeg, eds, "Approaches to qualitative research in mathematics education", Springer, Dordrecht, 2015, 365–380.

[**13**] H. Pollak, *The interaction between mathematics and other school subjects*, In: UNESCO, ed., "New trends in mathematics teaching IV", Paris, 1979, 232–248.

[**14**] L. Radford *The emergence of symbolic algebraic thinking in primary school*, In: C. Kieran, ed., "Teaching and learning algebraic thinking with 5- to 12-year-olds", ICME-13 Monographs, Springer, Cham, 2018, 3–25.

[**15**] S. Schukajlow, D. Leiss, R. Pekrun, W. Blum, M. Müeller and R. Messner, *Teaching methods for modelling problems and students' task-specific enjoyment, value, interest and self-efficacy expectations*, Educational Studies in Mathematics **79** (2012), 215–237.

[**16**] L. S. Shulman, *Those who understand: Knowledge growth in teaching*, Educational Researcher **15** (1986), 4–14.

[**17**] G. Schwarts, A. Coles and A. Arcavi, *Leading mathematics teacher discussions during professional development: challenges, opportunities, and discussion sense*, For the Learning of Mathematics **42** (2022), no. 2, 53–59.

[**18**] A. G. Thompson, *Teachers' beliefs and conceptions: A synthesis of the research*, In: D. A. Grouws, ed., "Handbook of research on mathematics teaching and learning", National Council of Teachers of Mathematics, Reston, Macmillan, New York, 1992, 127–146.

[**19**] G. Vergnaud, *La théorie des champs conceptuels*, Recherches en didactiques des mathématiques **10** (1990), 133–169.

Chiara Andrà
University of Eastern Piedmont
Dept. for Sustainable Development and Ecological Transition
Via Duomo 6
Vercelli, 13100, Italy
e-mail: chiara.andra@uniupo.it

Fracesca Martignone
University of Eastern Piedmont
Dept. of Sciences and Technological Innovation
Viale Teresa Michel 11
Alessandria, 15121, Italy
e-mail: francesca.martignone@uniupo.it