Riv. Mat. Univ. Parma, Vol. **14** (2023), 317-332

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Students' mathematical modelling competencies developed in addressing a real-world problem

Abstract. In the present work, the efficacy of an educational project on mathematical modelling, based on a real-world problem posed by a stakeholder, is evaluated using a qualitative analysis of the students' final outcome of the project, a scientific report. The analysis has been carried out based on the mathematical modelling competencies which can be retrieved in the report. Modelling competencies related to all the steps of the modelling cycle can be found, thus showing the efficacy of the project in making students going through the whole modelling process. Moreover, the analysis provided some indications for the characterization of one step of the modelling cycle in terms of competencies, which is not so much investigated in the literature. A scientific report can then be considered as an effective evaluation instrument for mathematical modelling teaching experiences based on real-world problems and carried out employing group work.

Keywords. Mathematical modelling, modelling competencies, modelling cycle, real world problem.

Mathematics Subject Classification: 97-02, 97C, 97M.

1 - Introduction and theoretical background

In the last decades mathematical modelling (MM) has been recognized as relevant both by scholars and educational policies for teaching mathematics [**1**,**5**]. MM is identified as a necessary tool to combine science and mathematics with citizenship education because it can help "to provide students with the necessary knowledge to make informed decisions, show the relevance of such knowledge for meaningful problems, and involve students in the practices of generating such knowledge" [**9**, p. 739].

Received: April 28, 2022; accepted in revised form: April 28, 2023.

As matter of fact, one of the most important purposes of schools is to promote the development by students of key competences to make them able to become active citizens, and MM may contribute in achieving such a goal [**5**, **9**]. This article focuses on mathematical competence, and, more precisely, on the mathematical modelling competency. We share the distinction proposed in [**11**] between mathematical competence and mathematical competency: mathematical competence is defined as "someone's insightful readiness to act appropriately in response to all kinds of mathematical challenges pertaining to given situations" [**11**, p. 12], while mathematical competency is "someone's insightful readiness to act appropriately in response to a specific sort of mathematical challenge in given situations" [**11**, p. 14].

Bringing MM within the mathematical curriculum is demanding both for teachers and students, so that in everyday mathematics teaching practice, there is still relatively few genuine modelling [**5**]. The main reasons are rooted on the fact that MM requires mathematical and extra-mathematical knowledge, as well as appropriate beliefs and attitudes, especially for more complex modelling activities [**5**, **11**].

Many researches (see [**1**] and the references within) concern about assessing and measuring teaching competencies for mathematical modelling, focusing on how mathematical modelling competencies can be assessed, and how assess modelling competencies for students and teachers [**7**]. This work frames within such context focusing on how to assess students' modelling competencies when they are exposed to real-world problems.

1.1 - *Mathematical modelling cycle and competencies*

The term mathematical modelling indicates the process of translating between the real world and mathematics in both directions [**4**]. More precisely, a mathematical model is a "deliberately simplified and formalized image of some part of the real world" ([**5**], p. 77), such as nature, society, everyday life and other disciplines. The purpose of building and making use of a model is to understand how to tackle problems related to the real world [**12**]. For our purpose, we focus on the modelling cycle [**5**, **6**], which is composed of seven steps within and between reality and mathematics (see Figure 1). The first two steps are within the reality and allow to move from the real situation to situation model (understanding) and to the real model (simplifying); the third step (mathematizing) bring to the mathematical model within the mathematics domain, where it is possible to work mathematically (step 4) reaching a mathematical result. The further two steps allow to go back to reality interpreting (step 5) and validating (step 6) the results. In the last step, the results (if

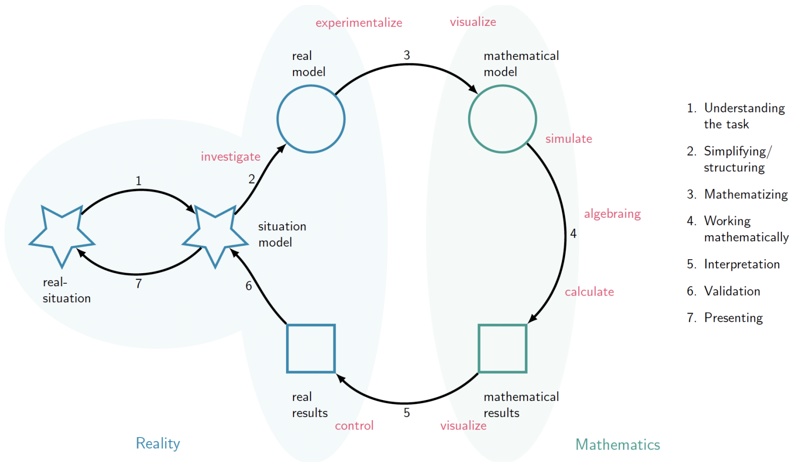


Fig. 1. Blum's modelling cycle with added influence of digital tools.

any) are presented in the real situation. Digital tools may play a crucial role in supporting learners facing the steps, for instance in step 4 students may resort digital tools to simulate, calculate and algebraing [**6**].

Teaching mathematics using MM means to carry out all steps of the modelling cycle and drive students through them. To achieve that, certain competencies or sub-competencies are required [**11**], for instance understanding a given real world situation or interpreting mathematical results in relation to a situation [**5**]. Modelling competency means the ability to construct, to use/apply mathematical models by carrying out appropriate steps as well as to analyze given models [**5**]. Within modelling competency it is possible to distinguish competencies related to different steps of the modelling process [**3**, **8**] as shown in Table 1. Nevertheless, step 7 "presenting" is not characterized by a specific set of competencies, so that we resort the "Mathematical communication competency-communicating in, with and about mathematics" defined as "an individual's ability to engage in written, oral, visual or gestural mathematical communication, in different genres, styles, and registers, and at different levels of conceptual, theoretical and technical precision, either as an interpreter of others' communication or as an active, constructive communicator" [**11**, p. 17].

2 - The educational project

The present study has the peculiarity that the didactical situation stems from a real-world problem posed by an actual stakeholder. The alderman to the culture of a small town wanted to involve the citizens to figure out how

Step	**Competencies**	**Label**
1, 2		
	to make assumptions for the problem and simplify the situation	$S_{1\text{-}2}.1$
	to recognize quantities that influence the situation, to name them and to identify key variables	$S_{1\text{-}2}.2$
	to construct relations between the variables	$S_{1\text{-}2}.3$
	to look for available information and to differentiate between relevant and irrelevant information	$S_{1\text{-}2}.4$
3		
	to mathematize relevant quantities and their relations	$S_3.1$
	to simplify relevant quantities and their relations if necessary and to reduce their number and complexity	$S_3.2$
	to choose appropriate mathematical notations and to represent situations graphically	$S_3.3$
4		
	to use heuristic strategies such as division of the problem into part problems, establishing relations to similar or analogous problems, rephrasing the problem, viewing the problem in a different form, varying the quantities or the available data etc.	$S_4.1$
	to use mathematical knowledge to solve the problem	$S_4.2$
5		
	to interpret mathematical results in extra-mathematical contexts	$S_5.1$
	to generalize solutions that were developed for a special situation	$S_5.2$
	to view solutions to a problem by using appropriate mathematical language and/or to communicate about the solutions	$S_5.3$
6		
	to critically check and reflect on found solutions	$S_6.1$
	to review some parts of the model or again go through the modelling process if solutions do not fit the situation	$S_6.2$
	to reflect on other ways of solving the problem or if solutions can be developed differently	$S_6.3$
	to generally question the model	$S_6.4$

Table 1. *Competencies related to the steps of the modelling cycle* [**3**, **8**].

young people use and what they would like to find in the municipal library. As a consequence, the stakeholder involved the local high school for addressing her question.

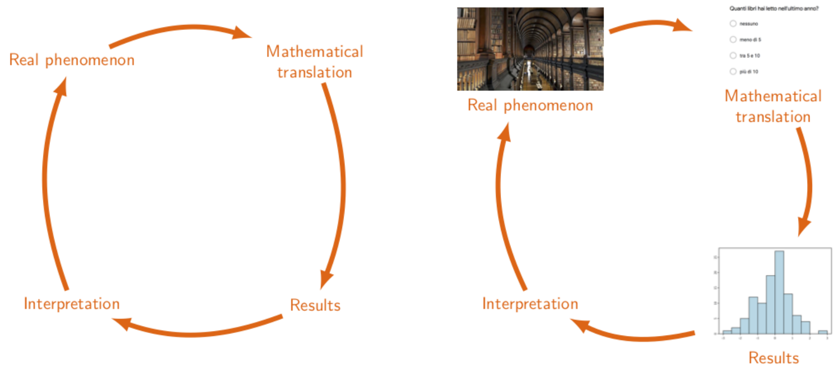
The educational project lasted six months, from January 2021 to June 2021, the researchers and the teacher planned eight meetings as milestones of the project. Each meeting lasted two hours and was held online by the researchers, while students attended the meetings from home and school according to the pandemic restrictions. The first three meetings, delivered as frontal lessons by statistics experts[1], aimed at providing some elementary statistical instruments, because the students had never been exposed to such a topic. Before the meetings the researchers, together with the statistics experts, analysed the real-world problem to provide to the students the statistical instrument they would have likely needed. The first two meetings concerned concepts about univariate and multivariate descriptive statistics, such as graphics (e.g., histograms, box plots, scatterplots, star plots) and indexes (e.g., location, dispersion, shape and correlation). The third meeting concerned inferential statistic and the introduction of the t-test. During the three meetings, the statistics experts gave also an insight into statistical applications, presenting some results of their own research. The remaining five meetings, delivered by the authors of the present paper, had the purpose to support students in doing the MM activities. We recall that the question posed by the stakeholder is an open-ended question which could generate in the students a feeling of "perplexity due to too many roads to take and no compass given" [**2**]. For this reason, the students have been guided through the modelling process by means of the specific **tasks** described in the following subsection. The meetings had a student-centered approach that is group work and classroom discussion, because they have proven to be particularly suited for modelling activity: they are able to activate students both cognitively and metacognitively [**5**] and they have a positive impact in terms of enjoyment, interest and self-efficacy [**13**].

In addition to the five meetings, the students worked in group, both autonomously and with the support of their teacher, beyond the meetings for approximately 80 additional hours. This was possible because the project involved the principal of the school and was framed within the PCTO (Percorsi per le Competenze Trasversali e l'Orientamento) program[2].

The question posed by the stakeholder (how young people use and what they would like to find in the municipal library) has been investigated by means of

[1]The statistics experts are researchers in statistics and teach elementary and advanced statistics at university.

[2]The PCTO program is a mandatory institutional internship that students have to attend as part of their education. The program allows collaboration with companies and academy.



(a) (b)

Fig. 2. (a) Simplified modelling cycle proposed in activity A0. (b) Simplified modelling cycle adapted to answer to the stakeholder's request.

a survey which has been built, successively delivered to a consistent sample of 1500 people (age ranging between 11 and 19 years old) and finally analyzed by the students using statistical methods and software. Then, the students wrote a scientific report which has been presented to the stakeholder. The last five meetings of the project were designed to turn students into mathematical researchers by means of six activities.

Activity A0. In the activity A0 (Introducing mathematical modelling), students were introduced to a simplified version of the modelling cycle, represented in figure 2a, in order to make them aware about the MM process, that positively influences the acquirement of modelling competencies [**8**]. Then, an example of modelling process has been presented and discussed with students. In particular, an animation movie (Kung-Fu Panda) has been modelled resorting the social networks tools and successively analyzed using two different centrality measures (degree and betweennes). Two different centrality measures have been introduced to show students the possibility to build different models for the same phenomenon and, thus, to show them the cyclic character of the mathematical modelling process. Then the following task, which has been successively carried out by means of a classroom discussion, has been delivered to the students: *A0.T1 - What could be the steps of a mathematical modelling process to help us answering to the alderman's question?*

Together with the students the simplified modelling cycle of figure 2a has been adapted to answer to the stakeholder's request (see figure 2b), putting the attention on the real phenomenon at hand (the young people's use of the library), its "translation" into mathematical language provided by the survey, the necessity for the interpretation of the results and, eventually, the re-design of the survey to better tackle the stakeholder's request.

Activity A1. The remaining activities were designed to allow students to go through all the steps of the Blum's modelling cycle. During activity A1 (Proposing the questions for the survey), first some indications for writing a survey have been provided to the students. In particular, students were exposed to examples of surveys, underling that the questions allow to explore different dimensions of a social phenomenon. Moreover, an example of a question whose answers could be biased has been given and the impact of the amplitude of the scale for the likert questions has been discussed. Afterwards, the students were asked to address the following tasks in small group: *A1.T1 - Propose some questions to be inserted in the survey. You can take inspiration from surveys to which you answered in the past or from the internet. A1.T2 - Identify which dimensions of the phenomenon "library" your questions aim at exploring and classify your questions according to such dimensions. A1.T3 - Discard the questions which are not relevant with respect to the alderman's request.*

Activity A2. As a result of A1, students proposed several questions labelled by their dimensions. With activity A2 (Formulating the questions/answer options for the survey) students were asked to address the following task: *A2.T1 - Break into groups. Each group should choose two different dimensions. Improve the questions related to these dimensions and, possibly, add further questions. Pay particular attention to make questions and answer options as clear as possible, to foresee and minimize possible biases, to carefully choose the scale for the likert questions, to choose suitable answer options.* A preliminary version of the survey was then submitted to a small sample (20 people). In this preliminary survey the students purposely formulated some questions as open questions, such that the answers could suggest possible answer options for the final version of the survey. Moreover, the students interviewed the participants to detect possible ambiguities and problems in answering to the survey.

Activities A3 and A4. Activities A3 (Using the statistics software R) and A4 (Analyzing the results of the survey) are strongly intertwined. In activity A3, students were introduced to the software R[3]. During the tutorial the students were asked to practice real-time with the software, using a given

[3]R-project is a professional statistical software.

dataset and, then, some data coming from the preliminary survey. While during activity A4, the students were asked to work in groups with the software R to analyze and to interpret the results of the survey. Activity A4 took one month and it was carried out by the students in groups under the supervision of their teacher.

Activity A5. Finally, in activity A5 (Writing the scientific report), an introduction about the structure and purpose of scientific reports has been delivered by the authors of the paper as a frontal lesson. A few examples have been provided to the students to clarify the role of the different sections which a scientific report should contain. Then, students were asked to work in groups on writing a scientific report to be delivered to the stakeholder. Groups worked on the different parts of the scientific report (introduction, methodology, results, conclusions). Also this activity took approximately one month and it was carried out by the students under the supervision of their teacher. Students worked on a shared document to build a draft version of the report (version V0). The authors of the present paper played the role of the reviewers, making remarks and suggesting modifications by directly commenting the shared document. The students then produced the final version of the report (version V1) after multiple reviews.

3 - Research questions

The main purpose of this work is to report an activity of MM focusing on the effectiveness of the experience with respect to the MM process. More precisely, we wonder to what extent the educational project activates MM competencies, and how teacher may evaluate the efficacy of such a project. In particular, we address the following research question: *RQ1. Which modelling competencies can be retrieved in the students' outcome of the project?* Moreover, due the lack of characterization of the "step 7, presenting", we also aim at addressing the following one: *RQ2. What are the competencies related to "step 7, presenting"?*

4 - Data gathering and methods of analysis

The participants of this study are 18 high school students (grade-11), 4 male and 14 female. The average grade of the class in maths is good (6.9 over 10), according to their last report card (December 2020). To address the above research questions, we analyzed the two versions of the scientific report written by the students, that is the draft (version V0) and the final version (V1), using qualitative content analysis [**10**]. More precisely, deductive coding has been applied to analyze sentences, paragraphs and plots of the report V1, using the MM competencies described in the section 1.1 as categories. Moreover, inductive coding on paragraphs and sentences has been employed to analyze the changes made by the students between reports V0 and V1. The categories which emerged from such an analysis allow to characterise the "step 7, presenting" in terms of competencies. All data were jointly coded by the authors.

5 - Data analysis and findings

In report V1, we can find the dimensions the students decided to investigate during activity A1 (Proposing the questions for the survey). Indeed, in the methodology section, the students listed the dimensions related to the stakeholder's question, as can be seen in the following excerpt:

> *[. . .] In particular we [students] decided to investigate the following dimensions: frequency of attendance, reason for attendance, the library services and how they are used, the affective dimension, reading habits, preferred literary gender and personal informations.*

In identifying such dimensions, the students "make assumptions for the problem and simplify the situation" ($S_{1\text{-}2}.1$) and "recognise quantities that influence the situation, name them and identify key variables" ($S_{1\text{-}2}.2$).

Students report different examples of questions they first proposed and they after decided to discard during activity A1: among them we find the question "Who approached you to reading?". They explain that they decided to discard this question being more related to reading habits than to the sample's relationship with the library. In doing this, students worked on competency $S_{1\text{-}2}.4$ (to look for available information and to differentiate between relevant and irrelevant information).

To investigate the dimension "library services", in a first proposal the students listed the different services and asked to the participants to the survey to choose the three most used ones. In order to better measure this dimension, the question was successively transformed, during activity A2 (Formulating the questions/answer options for the survey), in a likert scale question in which the participants were asked to rate each service from 1 (not at all) to 5 (very often), according to the frequency of use. By doing this, students first identified a key variable (frequency of use) for the dimension "library services" and, then, they decided how to measure it. Students worked on both competencies $S_{1\text{-}2}.2$ (to identify key variables) and $S_3.1$ (to mathematize relevant quantities and their relations).

Concerning the dimension “reason for attendance”, the students identified as a key variable the attractiveness of the local library with respect to nearby libraries. For this reason, students introduced a question to ask whether and why the participants to the survey frequented the library in a place different from their place of residence. Different reasons were listed, associated to a likert scale from 1 (I totally disagree) to 5 (I strongly agree). Also in this case students worked on competencies $S_{1\text{-}2}.2$ (to identify key variables) and $S_3.1$ (to mathematize relevant quantities and their relations).

In the report we can also find the description of the evolution undergone by some questions during activity A2. We consider as an example a question related to the dimension “personal informations” which aimed at investigating where the participants to the survey attend the library. In activity A1, the question was formulated as an open question (“Where do you go to the library?”). Then, during activity A2, the students realized the difficulties in analysing the results of such a question and they decided to turn the question into a closed question with three possible answers (1. Romano di Lombardia, 2. Other, 3. I don’t go to the library). In addition to the greater ease of analysis, this modification also better targets the stakeholder’s initial question (the stakeholder is the alderman to the culture of Romano di Lombardia). For the sake of clarity, the students decided to further modify the question as “Where do you go to the library more often?”, thus adding the frequency adverb “more often” in order not to excessively limit the possible answers. In realizing this process, students worked on competencies $S_{1\text{-}2}.2$ (to recognize quantities that influence the situation, to name them and to identify key variables), $S_{1\text{-}2}.4$ (to look for available information and to differentiate between relevant and irrelevant information) and $S_3.2$ (to simplify relevant quantities and their relations if necessary and to reduce their number and complexity). We can conclude that, during activity A1 and A2, students developed competencies related to “step 1, understanding the task”,“step 2, simplifying/structuring” and “step 3, mathematizing”.

The “Results” section of the report V1 provide the results of activities A3 (Working with the software R) and A4 (Analyzing the results of the survey). The students report the results of the computation of position indexes and shape indexes for quantitative variables (e.g., mean, median, skewness and kurtosis of the distribution of the number of books read in an year). In addition to these indexes belonging to univariate descriptive statistics, also the Spearman correlation coefficient was computed to correlate different variables (e.g., frequency of use of the online borrow vs library borrow). Moreover, the students provide different graphical representations for both quantitative (e.g., histograms and box plots) and qualitative (e.g., pie charts) variables, as suggested by the

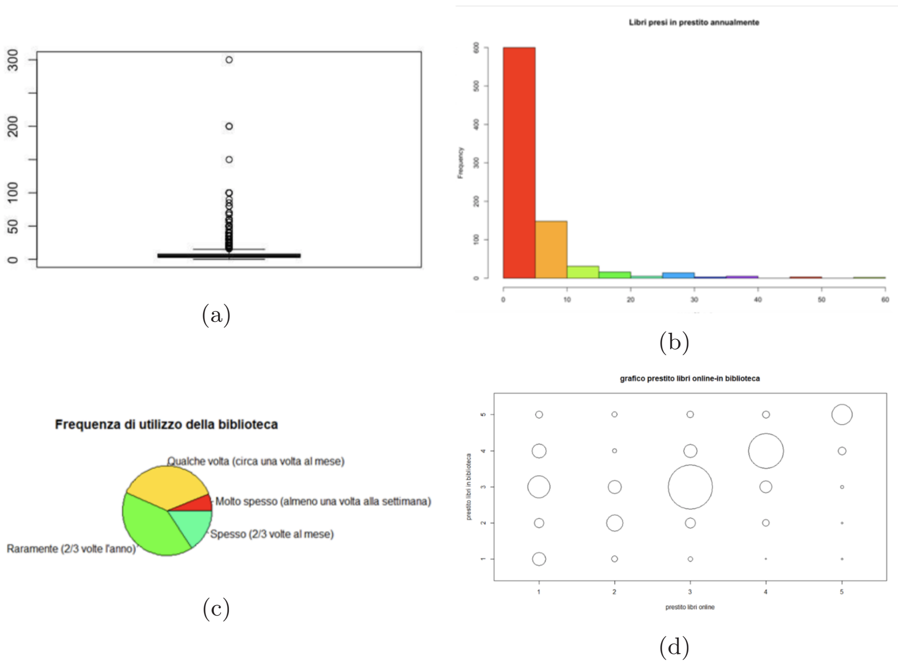


Fig. 3. Examples of plots built by the students: (a) Boxplot. (b) Histogram. (c) Pie chart. (d) Bubbleplot.

statistics experts in the first two meetings of the project. Figures 3a, 3b, 3c show examples of a boxplot, an histogram and a pie chart represented by the students in their report. The students also represent the correlation between the frequency of use of the online borrow and the library borrow, which have been previously measured with a likert scale question in which the participants were asked to rate each service from 1 (not at all) to 5 (very often), according to the frequency of use. In particular they employed a bubble plot, a scatter-plot in which the dimension of each point is proportional to the magnitude of the part of the statistic sample who chose the likert scale values corresponding to the coordinates of that point (see figure 3d). Notice that the students decided to use this plot, answering to a precise need, even if this type of plot had not been introduced by the statistics experts. Moreover the students use the t-test to compare the mean of different quantitative variables. Looking at the statistical analyses reported by the students, we notice that they worked on competencies $S_{1\text{-}2}.3$ (to construct relations between the variables), $S_3.1$ (to mathematize relevant quantities and their relations), $S_3.3$ (to choose appropriate mathematical notations and to represent situations graphically) and $S_4.2$ (to use mathematical knowledge to solve the problem). We can also retrieve competency S_5.1 (to interpret mathematical results in extra-mathematical contexts), as the following excerpts show:

> E1. *The collected data show that the library is seen by young people mainly as a place where to borrow books. It is appreciated also as a place where to study and where to take advantage of cultural activities, although to a lesser extent.*
>
> E2. *The waiting time to borrow a book is considered acceptable or short. This is probably due to the speed of the well developed inter-library borrow, which makes the exchange of books not physically present in a specific library very quick.*
>
> E3. *Especially the sub-sample of the visitors of the Romano di Lombardia library highlighted the necessity to improve and increase the number of the study stations.*

Looking at Figure 3a, we can notice the presence of some outliers which correspond to completely non realistic values for the number of books read in an year. Noticing this fact, the students decided to re-compute the mean of these data, neglecting the values greater than 100, thus working on compentency S_6.1 (to critically check and reflect of found solutions). Concluding, during activities A3 and A4, students worked on competencies related to "step 2, simplifying/structuring", "step 3, mathematizing", "step 4, working mathematically", "step 5, interpreting" and "step 6, validating".

To evaluate the competencies shown by students in activity A5 (writing the scientific report), we provide in the following some comparisons between the reports V0 and V1. In the report V0 not all key sections (introduction, methodology, results, conclusions) were present: as an example the "introduction" section was completely missing. Moreover, some parts of the report were inserted in a non appropriate section. An example is the following excerpt which was first introduced in the "results" section and therefore moved to the "methodology" section in report V1:

> E4. *It's important to highlight that, before realizing the analysis, a pre-processing of the data was necessary to make them analyzable: as an example fixing the typos in the year of birth, if possible.*

By doing this, students worked on a competency related to "step 7, presenting", which we name S_7.1 (to present the results with an appropriate structure according to the genre and style in which the results are presented).

In the following, the descriptions of the standard deviation in the "methodology" section from V0 and V1, respectively, are reported:

From V0: *Standard deviation: it corresponds to the mean distance of the points of the sample from their mean.*

From V1: *Standard deviation: it corresponds to the mean distance of the points of the sample from their mean. In the case of a data sample, it is computed as:*

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n-1}}.$$

The students explained that the choice to not insert formulas in the report draft was taken for the addressee of the report, the alderman to the culture, who might not understand the statistical details. The students after realized that the report wouldn't become clearer for non-statisticians by simply explaining the formulas in words and they decided to re-insert the mathematical formulas while, at the same time, they paid attention not to write a too technical "conclusions" section, such that the main results could be understood also by non-experts in statistics. Also in this case to characterize the process the students engaged in, we propose a competency related to step 7 which is $S_7.2$ (to present the results in a way which is appropriate for the addressee). The "conclusions" section has been strongly modified by the students from version V0 to V1 of the report. In particular, in the version V0, some statements which were not supported by the data were reported by the students, as in the following excerpt:

E5. *Concluding, it was interesting to discover that a strong social utility is attributed to the service provided by the library: the library contributes to spread culture also among those with poor financial means.*

In the report V1 statements, like the E5, were removed, and students reported claims strongly supported by data. From these results, we propose $S_7.3$ (to present the results in a rigorous way, in particular to present statements strongly supported by data) as a competency to further characterize step 7.

Students upgraded the version V0 of the report also from the formal point of view, like, for example, correcting grammar mistakes and numbering tables and figures, thus working on a competency which we name $S_7.4$ (to present the results in a clear and correct way from the formal point of view).

6 - Discussion and conclusions

According to the results presented in the previous section, we can answer to RQ1 (Which modelling competencies can be retrieved in the students' outcome of the project?) arguing that MM competencies related to the first six steps of the modelling cycle can be retrieved in the students' final report, showing that the students went through these steps. We consider this fact as an indicator of the effectiveness of the educational project in making students work on MM competencies.

The results also confirm that the students progress in MM in a non-linear way [**5**], jumping back and forth between the steps in the MM cycle: for example, while in activity A1 and A2 we can retrieve competencies related to "step 3, mathematizing", in activity A4 and A5 students take "a step backward" and we are able to find competencies related to "step 2, simplifying/structuring".

Concerning RQ2 (What are the competencies related to "step 7, presenting"?), we have identified competencies related to the step 7 of the MM cycle which can be framed within the definition of "Mathematical communication competency- communicating in, with and about mathematics" [**11**]. More precisely we have identified the following four competencies:

S_7.1 to present the results with an appropriate structure according to the genre and style in which the results are presented.

S_7.2 to present the results in a way which is appropriate for the addressee.

S_7.3 to present the results in a rigorous way, in particular to present statements strongly supported by data.

S_7.4 to present the results in a clear and correct way from the formal point of view.

We are aware that these competencies stem from a specific student outcome, that is a scientific report. Nevertheless, we think that they are not format dependent and, thus, they can be used to analyse also other forms, genres and styles of student's outcome.

To conclude, concerning how to assess students' MM competencies [**7**], we argue that, when a real-world problem is addressed by students working in groups, a scientific report is an effective evaluation instrument for the assessment of the students when going through a MM process. In a future work we plan to overcome the limitations concerning the joint coding procedure and the sample size by carrying out a systematic study of students' productions in the framework of a larger project involving 30 classes from all over Italy.

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