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A Study and Research Path about the evolution of COVID-19 at secondary school: Conditions for an interdisciplinary approach

Abstract. This paper discusses the experience of the design and implementation of an instructional proposal about the evolution of the COVID-19 pandemic in secondary school. The proposal is based on the so-called 'study and research paths' (SRP) grounded on the Anthropological Theory of the Didactic. This SRP is closely linked to the design of a teacher education proposal (described in [**1**]) aiming to provide secondary school teachers with tools to address interdisciplinarity. We here focus on the collaboration between researchers and secondary school teachers in adapting and transposing an open teaching project to inquire into the role of models and modelling from different disciplines in the study of the evolution of COVID-19. We focus on discussing the ecology of the 'interdisciplinary' SRP, that is, the analysis of the conditions that have facilitated the development of this SRP and the constraints that hinder its progress within an interdisciplinary approach.

Keywords. Interdisciplinarity, Study and Research Paths, anthropological theory of the didactic, secondary school level, modelling, pandemic.

Mathematics Subject Classification: 97M10, 97C70, 97B10.

1 - Introduction

Research in mathematics education has recognized the importance of including applications and mathematical modelling in the teaching and learning

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of mathematics [**3**]. In this respect, we have several examples of major issues in our society that require a collective scientific effort working across the boundaries of the scientific disciplines, where mathematics and mathematical modelling can be seen and act as service subjects.

Besides all the progress made in research and the support of educational policies and curriculum reforms, well-established modelling activities do not disseminate as expected. They confront critical constraints that hinder their long-term "survival", some of them related to the difficulty of treating interdisciplinarity contexts in schools. These constraints are connected to important interrelated didactic phenomena that exist in school institutions. One is the isolation of disciplines and the prevalence of monodisciplinary curricula [**9**]. Another one is the dominant way to organize the teaching and learning of school disciplines, based on the logic of (monodisciplinary) concepts rather than the logic of (multidisciplinary) problems. Finally, therefore, there is a lack of epistemological and didactic tools to approach modelling in the interaction among disciplines [**10**]. The discussion about interdisciplinary education is connected to the issue of STEM education [**8**] from three different perspectives: twenty-first-century skills, mathematical modelling, and education for responsible citizenship. [**7**] makes the role of modelling in STEM education clear: "modelling is a powerful vehicle for bringing features of 21st-century problems into the mathematics classroom".

During the COVID-19 pandemic, more than ever, citizens and, in particular, students have been exposed to and asked to understand how mathematical and scientific advances contribute to the comprehension of societal phenomena, in this case, related to the evolution of the pandemic. Mathematical models have been widely used and disseminated to analyze the spread of the virus and models have been often employed to guide policy decisions in handling the COVID-19 crisis. As expressed by [**11**] in their manifesto about the "Five ways to ensure that models serve society":

"Mathematical models are a great way to explore questions. They are also a dangerous way to assert answers. Asking models for certainty or consensus is more a sign of the difficulties in making controversial decisions than it is a solution, and can invite ritualistic use of quantification. Models' assumptions and limitations must be appraised openly and honestly. Process and ethics matter as much as intellectual prowess. It follows, in our view, that good modelling cannot be done by modellers alone. It is a social activity" (Op. cit., p. 484).

In the context of the IDENTITIES project, the issue of the role of models, modelling and simulation has been one of the central topics addressed in the design of an instructional module for teacher education about interdisciplinarity (see [**1**], this issue). This module was tested in a local implementation during 4 sessions of 3 hours in the academic year 2021-22 at the University of Barcelona and in an international IDENTITIES Summer School (in 2021 and 2022, for about 10 hours), where the first two authors were involved, as a participant and as an educator, respectively. The teacher education proposal focused on modelling activities that could be transposed to secondary schools to address live societal questions that emerged during the scientific approach to the pandemics. Along with the different submodules of the proposal, participants were asked to assume different roles to facilitate questioning together (teachers and educators) on the way to describe, analyze and design possible modelling activities. In the initial submodules, teachers-participants were asked to experience as "students" a teaching project, a study and research path (SRP), previously designed by the researchers linked to the role of models, modelling and simulations in the study of the evolution of the pandemics. The following submodules aimed at introducing and transferring epistemological and linguistic tools to teachers-participants for the analysis of interdisciplinarity. In the last module, teachers in training worked on the design of an adaptation of the experienced SRP, and, in case they had the chance, they implemented it in a real secondary school context. This paper focuses on the work developed by one of the participants, the first author of this paper, a secondary school teacher and researcher in didactics of mathematics.

We focus on the experience with the design and implementation of an SRP about modelling the COVID-19 evolution for secondary school education, with the collaboration of teachers (non-researchers) of different subjects (mathematics, biology, and english). This particular SRP has been implemented twice, in April-June 2020, with the beginning of the pandemic, and in February-March 2021. We are interested in different aspects that emerged from this experience, which include stages of design of an interdisciplinary SRP, in collaboration between researchers and non-researchers, and stages of analysis of the implementation, we are interested in different aspects. On the one hand, we want to focus on the *conditions* that facilitate the collaboration between teachers, and students, to initiate and guide an SRP with a fluent interaction among different disciplines; on the other hand, we want to analyze the *limitations* or *constraints* that hinder its progress toward a more prosperous interdisciplinary approach.

2 - Research Framework and Methodology

Within the framework of the Anthropological Theory of the Didactic (ATD), the main one used in this paper, a change of school paradigm is proposed to overcome some of the main didactic phenomena linked to the "monumentalisation" of the taught knowledge [**6**]. This change has been described in terms of a paradigm shift, from the "paradigm of visiting works" to the "paradigm of questioning the world". Chevallard characterizes the transformation in mathematics education not only at the pedagogical level ("how to teach?") but also at the didactic and epistemological levels, dealing with the question about "what and how to teach?". In the paradigm of questioning the world, the knowledge to be taught is associated with the inquiry of relevant questions. Approaching these questions includes moments of study (searching for available answers in the media) and moments of inquiry (deconstruction and reconstruction of knowledge to generate one's answer), leading to what is known as a *study and research path* (SRP) [**4**, **6**, **12**]. Carrying out an SRP starts with the consideration of an open *generating question* Q_0, which involves raising derived questions Q', Q'', etc., searching already available pieces of answer or knowledge tools, mobilizing knowledge, and other kinds of resources to validate, adapt, and develop the information found. Compared to the traditional notion of knowledge in school institutions, implementing question-led study processes helps the knowledge to be taught to become dynamic, provisional, and collective. It also generates interdisciplinary processes because questions are often raised in social contexts without being associated with a single discipline. Finding the disciplines that can provide productive answers to the questions addressed as part of the inquiry process, as it is necessary to merge the tools obtained in different disciplines or domains for the elaboration of a final answer to Q_0.

This paper presents an example of a modelling project about the COVID-19 evolution conceived as an SRP. Our research methodology corresponds to the *didactic engineering process* [**2**] structured in four steps. First, is the identification of didactic phenomena to address. In our case, they correspond to the consequences of school isolation of disciplines and the difficulties that this isolation creates in the approach of open questions that are not raised within a single discipline. The second step refers to the *a priori* analysis of a given teaching proposal under certain conditions: here, it is the design of an SRP about the COVID-19 evolution. The implementation of the SRP appears as the third step or *in vivo* analysis, to gather information and evidence about the implemented didactic process. Finally, the fourth step corresponds to *a posteriori* analysis that goes back to the conditions established for the running of the SRP, its design and the didactic phenomena at stake. In the next sections, we use some of the main traits of SRPs, in particular:

- The starting point of an SRP is a generating question Q_0 posed by the teacher(s) and addressed to the community of study: the students and the teacher(s). Neither teacher(s) nor students need to know the answer to Q_0 in advance. There are generally different possible types of final answers to Q_0. What needs to be clear at the beginning is to whom the final answer is addressed and how it will be received and assessed.

- The study community addresses the generating question by opening many derived questions and proposing partial answers to these questions. An arborescence of questions and answers – a *questions-answers map* [**12**] – is used to describe the possible paths to follow (*a priori* design) or those actually covered (*in vivo* or *a posteriori* analysis). Questions-answers maps (Q-A maps) play a key role in the management of the SRP because they help provide a disciplinary-neutral terminology to describe the paths followed and conceive the new lines of the inquiry at different moments of the process.

- Running an SRP includes searching for available answers in the *media* (the internet, textbooks, articles, experts, etc.) and testing their value and productivity to answer Q_0 or the derived questions Q', Q'', etc. Knowledge tools of different natures and disciplines are mobilized not because of their importance or domain but for their utility to answer the questions raised. The study and testing of the searched information and knowledge tools usually leads to the raising of new derived questions, thus producing a self-sustained process [**5**].

3 - Conditions for the design and implementation of an SRP about modelling COVID-19 evolution in secondary school

The SRP about modelling the COVID-19 evolution has been implemented twice, in April-June 2020 with the beginning of the pandemic (see [**13**]), and in February-March 2021. Due to the exceptional conditions of the first implementation, this paper focuses on the second edition as its design was improved and the conditions for implementation were more stable (at least, than during the pandemic lockdown). The implementation was carried out at Col.legi Natzaret, in Esplugues de Llobregat, a town near Barcelona, with 60 students of grade 10 (15-17 years old) distributed in two parallel groups. It was developed as an interdisciplinary project involving the subjects of mathematics, biology, and oral and written expression. Students were organized in working teams of six members, with heterogeneity in relation to their academic performance.

The SRP ran over 17 one-hour sessions during the official hours of mathematics, biology and oral and written expression. It ran under relatively regular conditions, although the limitations due to the pandemic: the parallel groups could not interact, and each teacher was assigned to only one of the groups. Four teachers participated in the implementation: two mathematics teachers (one being the first author of the paper), a biology teacher, and an english teacher taking care of the course on communication skills. Both teachers, the biology and english ones had half of the students in the corresponding optional subjects. In collaboration with her research team, the first author developed the *a priori* design of the SRP. The rest of the teachers had no direct involvement in the design. Still, they got actively engaged in deciding how to present the project to students and in the *in vivo* analysis during its implementation. Some special sessions were organized with all the teachers to agree on how to introduce the project, the timing, the way to distribute the students and the strategy to manage the SRP. Then, during the implementation, the teachers shared a journal where they daily reported their work with the class and the teaching materials (their presentations, students' reports, evaluation criteria, among other aspects).

Students worked collaboratively with online and digital tools. The teachers used a shared google presentation to report the progress of each working team. In addition, after each session, the working teams worked with the same template to document the advances of their inquiry. They had to report on the questions they had addressed, the temporary answers found, the tasks developed individually and in groups, and the new questions to follow with. Besides these shared documents, students had access to a presentation with some common instructions, indicating what was expected from their work and the steps to follow. From the start, the students were informed that they were responsible for defining the questions to address and the hypothesis they had about pandemic evolution. They had to update their questions-answers map regularly and, in the end, prepare an informative video presenting the results from their research to be distributed to the school community. The SRP teachers evaluated the students' presentations, with some invited teachers from other subjects. The final assessment also included the other resources produced during the SRP, like the logbooks and the Q-A maps.

4 - Results of the experienced SRP about COVID-19 evolution

4.1 - *Transferring the responsibility to formulate "Researchable" questions*

The generating question Q_0 students were asked to confront with was a general extra-mathematical problem of particular social relevance due to the excess of news related to the pandemic. They were asked to run an awareness campaign for the school about the pandemic and its impact on society. They were responsible for providing contrasted and scientifically founded information and defining what they wanted to address. The final answer must take the form of a short video presentation.

In the beginning, students were told to approach their research from three complementary points of view: from the *available data* (accessible through the Spanish government website) and the *mathematical models* they could use (Which data may be selected to understand the evolution of the pandemic? How can mathematics and mathematical models help us to understand the evolution of the pandemic?); from the *biological knowledge* of the disease (How is the virus behaving?); and from the *societal impact* of the pandemic (What impact and effects are the pandemics having on our society?). Students were asked to delimit their focus by always keeping in mind these three complementary general questions.

Students began by gathering the concerns of the educational community, starting with their own and surveying their classmates and families. This helped them define the questions they wanted to address and plan the first steps of their inquiry. At the end of these first steps, each team had to present the general topic and identify three interrelated "researchable" questions concerning the mathematical, biological and societal aspects. Some examples of the researchable questions they posed are: How long does COVID-19 survive on a surface? What are the characteristics of the virus that make it so deadly? What age groups are the ones more affected? What are the physical sequelae of the disease? About the societal questions, examples of the ones proposed by the students were: How has the pandemic affected tourism in Barcelona? How has confinement affected people's daily lives? What different restrictions were implemented in Madrid during the three waves in comparison to Barcelona? Concerning the mathematics questions, those with a descriptive nature were more frequent: Which autonomic communities in Spain have been more affected? How can we measure if the first wave was worse than the rest? Are there important differences between the evolution of the case numbers (infected, death, recovered) over the two consecutive years? However, there were also some groups that included questions about the evolution of the data: How has COVID evolved in Catalonia? How has it evolved in different counties?

As it can be read in the project presentation,[1] the whole implementation followed three main phases. The first phase includes the (a) generation and formulation of "researchable" questions, (b) exploration of databases, and (c) presentation of specific questions and hypotheses to address. The second one focuses on (a) looking for and organizing the most relevant data for their inquiry, and (b) analyzing data and proposing models to fit data and predict the evolution of the pandemics. The third and last step corresponds to the students' elaboration of an informative video. In the following section, we focus

[1] The project presentation is available at https://bit.ly/3tRQpPz

on the student's work in the first two phases, paying special attention to the researchable questions with a mathematics intervention.

4.2 - *The role of questions-answers maps for students and for teachers*

During the sessions guided by the mathematics teachers, the different teams addressed their researchable questions. To facilitate their work, the mathematics teachers asked them to follow a structured report. On the one hand, each team had to make explicit the main questions they wanted to address, their hypothesis or preliminary answers, and the data they worked with. On the other hand, they had to fill out their *map of questions and answers* to describe the particular study and research trajectory they were following. This device, which was used during the whole implementation, took a crucial role for several reasons. First, it allowed students to have a common instrument for all the sessions and make explicit the evolution of the inquiry. Second, it helped teachers from different disciplines to follow the work of the teams. Moreover, students used this organization to address the questions of each discipline with the corresponding teachers. Figure 1 shows an example of *Team A*, which is commented on below. Additionally, at the end of the implementation, the assessment of these maps consider the completeness and classification of all the elements, the relevance of the questions, their creativity and accuracy.

Five sessions were devoted to the first phase of the project: to delimit and construct the system. During these sessions, students were provided with a database from the Spanish ministry that regularly updated the data about the evolution of the pandemic since its beginning. Students found different spreadsheets with accumulated data on cases, deaths, and ICU admissions. These worksheets also included information by sex, age groups, provinces, and communities. This large amount of data created important limitations. On the one hand, they had to be very careful in defining what they were interested in looking at, that is, to delimit and construct the *system* – that is, the part of reality that is to be modelled –, as well as the particular questions they wanted to address. That is why the teachers were especially attentive to helping them in delimiting the system by selecting the variables to consider, formulating the initial hypothesis to contrast, etc. On the other hand, they needed to learn some techniques to work with Excel to manipulate big spreadsheets easily. They had some experience with Excel but as beginners users. Then, the mathematics teachers had to dedicate some common sessions to respond to these necessities. For instance, students asked how to sort a list of data by value, and how to filter by defining some criteria (e.g., provinces or age groups), among other utilities.

In the particular case of *Team A*, they were first interested in this initial question $Q_{0-TeamA}$: "*Which has been the 'worst' wave of the pandemics in Spain? Has the second wave been worse than the first one (as said in the media)?*". In these first steps, they started to define what they wanted to address (length, number of infections, hospitalization and death, in global and by different groupings):

- Q_{1dates}: How long did each wave last?
  - $Q_{1.1}$: When do we start counting the beginning of a wave and its ending?
- $Q_{2infections}$: How many infections were there in each wave?
  - $Q_{2.1}$: How many deaths by sex were there? In total? By sex? By age group?
  - $Q_{2.2}$: Which were the most infected age groups?
    - $Q_{2.2.1}$: How many infected people were between 0-9 years old? 10-19? 20-29?...
- $Q_{3hospitalizations}$: How many people were hospitalized during each wave?
  - $Q_{3.1}$: In terms of hospitalizations, which wave has been the worst?
- $Q_{4deaths}$: What is the number of deaths in each wave?
  - $Q_{4.1}$: How many deaths by sex were there? In total? By sex? By age group?
    - $Q_{4.1.1}$: What can explain why men seem more likely to die?
    - $Q_{4.1.2}$: Does the growth rate of mortality have the same trend for each group age?

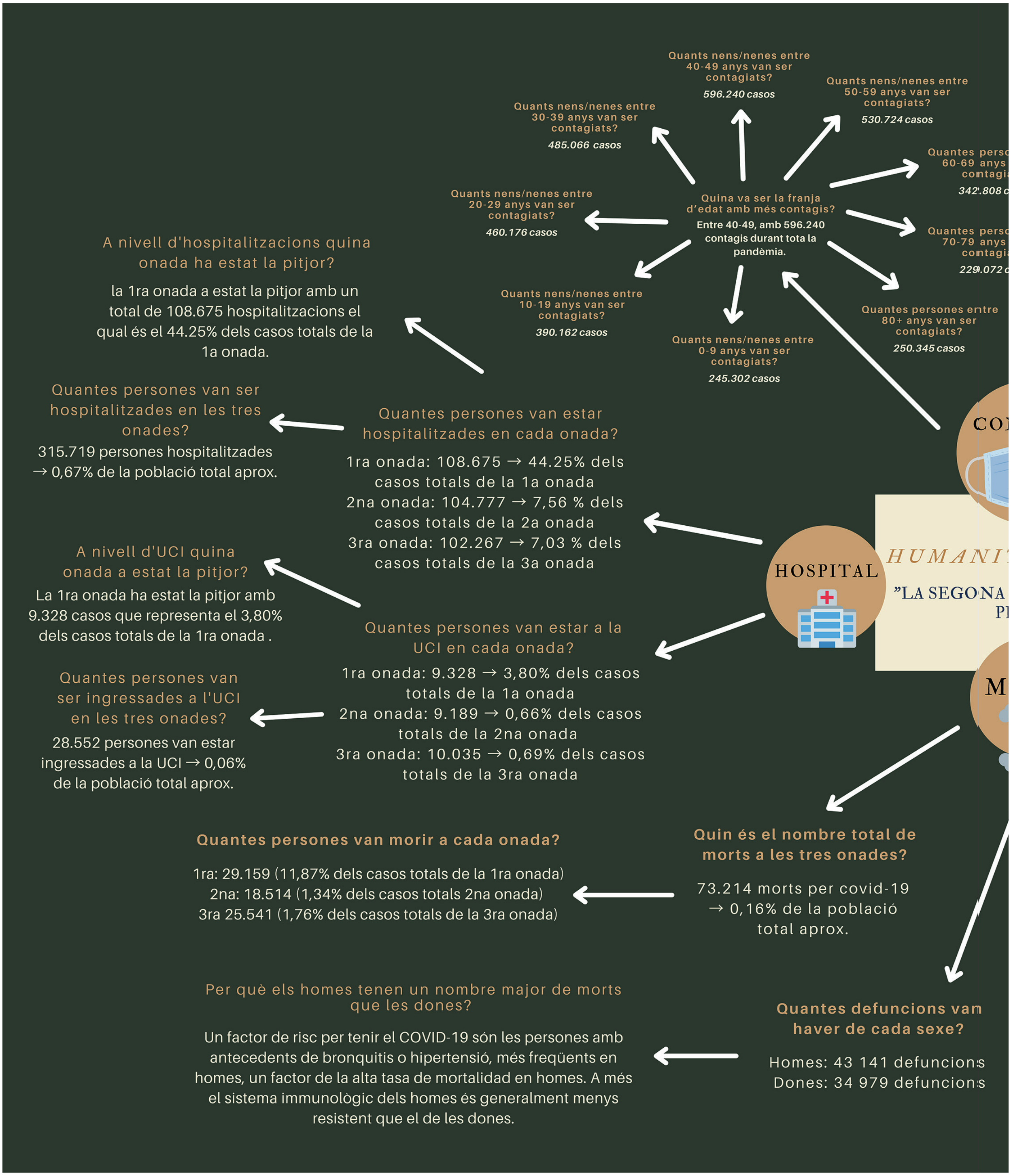
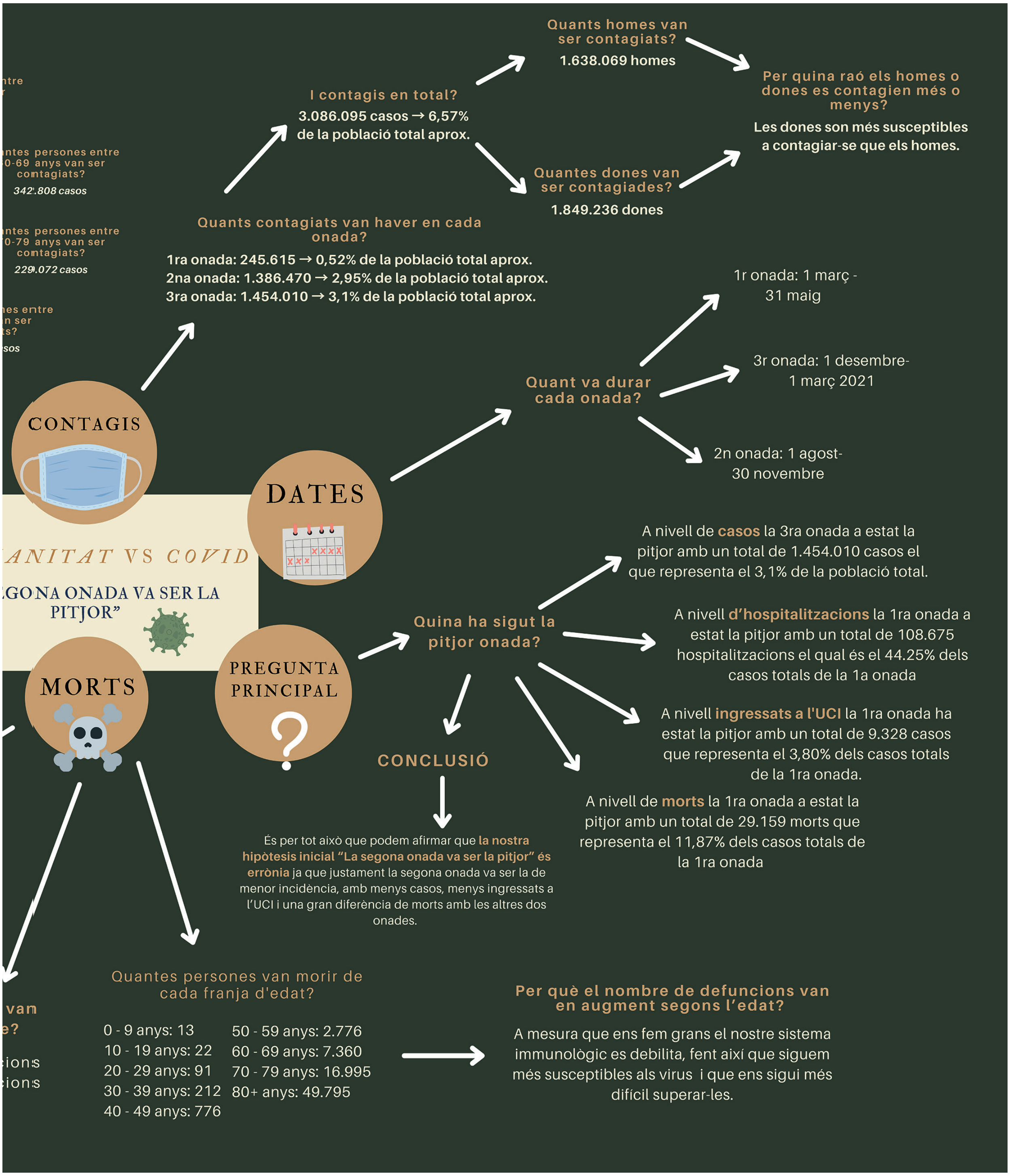


Fig. 1. Example of Q-A map developed by *Team A*.


CONTAGIS
DATES
MORTS
PREGUNTA PRINCIPAL
I contagis en total?
3.086.095 casos → 6,57% de la població total aprox.
Quants homes van ser contagiats?
1.638.069 homes
Quantes dones van ser contagiades?
1.849.236 dones
Per quina raó els homes o dones es contagien més o menys?
Les dones son més susceptibles a contagiar-se que els homes.
Quants contagiats van haver en cada onada?
1ra onada: 245.615 → 0,52% de la població total aprox.
2na onada: 1.386.470 → 2,95% de la població total aprox.
3ra onada: 1.454.010 → 3,1% de la població total aprox.
342.808 casos
229.072 casos
Quant va durar cada onada?
1r onada: 1 març - 31 maig
3r onada: 1 desembre- 1 març 2021
2n onada: 1 agost- 30 novembre
PITJOR”
Quina ha sigut la pitjor onada?
A nivell de casos la 3ra onada a estat la pitjor amb un total de 1.454.010 casos el que representa el 3,1% de la població total.
A nivell d’hospitalitzacions la 1ra onada a estat la pitjor amb un total de 108.675 hospitalitzacions el qual és el 44.25% dels casos totals de la 1a onada
A nivell ingressats a l'UCI la 1ra onada ha estat la pitjor amb un total de 9.328 casos que representa el 3,80% dels casos totals de la 1ra onada.
A nivell de morts la 1ra onada a estat la pitjor amb un total de 29.159 morts que representa el 11,87% dels casos totals de la 1ra onada
CONCLUSIÓ
És per tot això que podem afirmar que la nostra hipòtesis inicial “La segona onada va ser la pitjor” és errònia ja que justament la segona onada va ser la de menor incidència, amb menys casos, menys ingressats a l’UCI i una gran diferència de morts amb les altres dos onades.
Quantes persones van morir de cada franja d'edat?
0 - 9 anys: 13
10 - 19 anys: 22
20 - 29 anys: 91
30 - 39 anys: 212
40 - 49 anys: 776
50 - 59 anys: 2.776
60 - 69 anys: 7.360
70 - 79 anys: 16.995
80+ anys: 49.795
Per què el nombre de defuncions van en augment segons l’edat?
A mesura que ens fem grans el nostre sistema immunològic es debilita, fent així que siguem més susceptibles als virus i que ens sigui més difícil superar-les.

These questions mainly correspond to delimiting the system and representing the data numerically and graphically. The same happened with the rest of the groups, who mostly worked on the graphical representation of the data (once selected and manipulated). For groups who posed some questions about the pandemic evolution, the most common was the graphical representation of data concerning time. For instance, here is a sample of the questions posed by *Team B*:

- Q_{0TeamB}: Which wave has affected Madrid the most?
  - Q_1: Which wave shows the highest speed in the increase in the number of cases? And less?
    - $Q_{1.1}$: What is the date with the most cases registered?
    - $Q_{1.2}$: How long has each wave lasted?
    - $Q_{1.3}$: Why are there fewer registered cases on the weekends?
    - $Q_{1.4}$: How do restrictions affect the registration of cases?
  - Q_2: Which wave has the highest number of cases, regardless of its duration?
  - Q_3: Which wave presents the highest number of cases, considering its duration?

This team wanted to analyze the variation in the number of cases. They defined and calculated the *speed* of the number of cases using variation taxes. However, none of them started to propose equation-based models to fit the data and forecast what can happen in the future. All the groups posed questions to describe the data or compare them by age or place, but none of them tried to go further.

In the last session, the teams made presentations to their peers. One of the key instruments that teams shared with the rest was the Q-A maps that synthesized their inquiry. Interesting ideas emerged from the comparison of the different paths, such as: Which is the correct way of comparing the data from two or more provinces? Which tools exist and are more understandable to analyze the variation of data (rates of growth)? Which can be the interpretation of the increase or decrease, speed of increase or decrease of data? Does it make sense to talk about linear or exponential growth?, among other questions that could guide the continuation of the SRP into more advanced mathematical tools.

If we focus on the teachers' work guiding the SRP, it is important to mention that the Q-A maps elaborated by the different working teams have been recognized also by teachers as an important tool facilitating the guidance of the project and the collaboration among teachers from the different disciplines. On the one hand, the Q-A maps (new for some of them) helped them to monitor the work of each team (see comments from Teacher B), due to the openness of the initial researchable questions chosen and the diversity of paths followed. Moreover, students used these maps to organize the questions selected. Students, but also teachers, used them to plan when and how to address some of the questions (in what subject, with the whole class or with some of the working subgroups, ...). Additionally, at the end of the implementation, the assessment of these Q-A maps was considered as one central aspect, assessing relevant aspects such as: its completeness to describe the path followed, the classification and hierarchical organization proposed of all the elements (e.g. questions, answers, terminology, strategies followed).

With this respect, during the interviews realized with the teachers, after the implementation, the second mathematics teacher (Teacher B) explained that:

"[...] The map of questions and answers has a double function. The first is to see how the teams integrate the different questions that emerged during their inquiry; and, then, to synthesize what they knew before and what they know after the process they have followed. It is a good tool for monitoring, to know which group is more or less lost [...]."

Or, in the case of the teacher of biology, Teacher C:

"[...] It [the Q-A maps] offers double work for students: along with their research, students find answers (outside and inside the class), then they have to know how to pose, how to write the question(s) to which you have already given the answer. So they (the students) are responsible for finding the coherence between what they put as a question and what they write as an answer."

"[...] In the map you [referring to the teacher] can see the progress because it is where you see that there are questions that have answers. [...] It is a tool for monitoring the student's learning and the group's process."

5 - Discussion and conclusions concerning the conditions and constraints for the implementation of an interdisciplinary SRP

From the implementation of the SRP about the COVID-19 evolution different positive aspects emerged. First, the topic and the initial questions posed by the students were clearly of social relevance and the utility of the final answers requested was beyond doubt. The interdisciplinarity character of the questions and their treatment was also visible, as we all saw during the pandemics in the discussions presented in the media, showing different controversies between scientifics' and policy-makers' decisions, and also among experts of different areas (medicine, mathematics, economics, psychology, etc.). Second, the school offered special positive conditions to run the SRP. Having the possibility to organize a common teaching process with grade 10 teachers of three different subjects is not always possible. And having their acceptance to participate needs the implicit support and encouragement of the head teacher and the school management team. The choice of the SRP as an instructional proposal and the tools provided by the ATD to conceive modelling processes facilitate putting the questions addressed at the core of the inquiry process without "caring" about the type of knowledge tools that are used to elaborate the final answers. Finally, the teacher who assumed the responsibility of the SRP is an expert in the ATD, she has been implementing this type of instructional proposal for three years and, therefore, has certain expertise in its design and management. Also, the students had previously followed an SRP during grade 9 (see [**14**]) and were used to teamwork and project-based learning in all the subjects.

However, having apparently "good" conditions for the running of an interdisciplinary process does not mean that disciplines will interact in the elaboration of the final answer to the SRP. Indeed, also important *constraints* appeared that hinder this interaction and to which more attention should be paid in future research. The most important one is the teachers' *disciplinary profile* and their resistance to guiding the students' work unless the questions approached were clearly related to their discipline. Another one concerning the scholar organization level: among the three subjects that were merged to run the SRP, mathematics was compulsory for all grade 10 students, but biology and communication, on the contrary, were elective. Therefore, some students dedicated more hours than others to the development of the project. In the same way, the mathematics teachers spent 4 hours per week and the teachers of the optional subjects only 3 hours per week. This was an important constraint for both teachers and students, who were unequally involved. It also caused hierarchies in the project production and sometimes resulted in students not being able to participate as they feel disconnected from their team's progress.

Moreover, teachers highlighted the lack of time available as one of the big constraints that prevented them from more effective coordination. As explained by the biology teacher (Teacher C) during the interview:

"[We] have two meeting hours, which are where teachers are supposed to work on everything that being a teacher entails [...]. For the projects to be successful and enriching, they require a good design and presentation, but it is also necessary time to attend students. It would even be perfect to do a second round of the project: once the students have presented their work, the teachers should be able to meet and comment on it [...]. These hours do not exist, even if we ask for them. Instead of this, they [the school] introduce new "fashionable" methodologies (without foundation). With this, we are preventing ourselves from making good use of a methodology that has good potential [...]."

Following similar arguments, the communication teacher (Teacher D) claimed that:

"This project has been made during our free hours, while we were having lunch, [...]. I'm sorry we can't work more with teachers from other subjects because the school does not give us the time, it's impossible. [...] The school encourages us to do this type of implementation, but they do not give us the resources or working time to do it well."

What teachers missed was to have time to share and reflect on what had happened in class and to coordinate their intervention according to the students' work. Consequently, teachers were mainly aware of the student's work did during their class hours, but under their advice, as experts in some of the intervening disciplines. That is, due to the school organization, the teachers of the different disciplines (mathematics, biology and English/communication) could not be at the same time guiding the SRP. This has consequences because students tended to choose questions to address that seemed closer to the subject or teacher's disciplinary domain. For instance, in the case of Teacher D, the English teacher:

"[I] felt that I had been able to follow the biology and social research closely, as this was what the students were working on when the biology teacher and myself were in the classroom [with independent groups]. During our sessions, the students did not work on the mathematics questions, as the mathematics teachers were not in the classroom."

It was at the final presentation that she was able to see the result of the mathematics research. She also commented that she was able to understand the mathematics part of the research because it was explained at a simple level.

Interdisciplinary work needs quality time for the teachers to share opinions and make decisions. It has to overcome a long tradition of secondary school mono-disciplinary treatment of topics. Moreover, it needs time for teachers (and students) to think together about the necessary knowledge that is on the frontiers between the disciplines and the new knowledge coming from the interaction and integration of disciplines."

With this respect, teachers and researchers must collaborate during the SRP implementation – the *in vivo* analysis – to detect where and how disciplinary and interdisciplinary knowledge is needed. And this collaboration needs a previous collective reflection on what it means to design and guide an SRP coordinating and integrating different disciplines. Teachers participating in this implementation refer to how conscious they were about the project's multidisciplinary nature. However, this does not mean that they knew how to deal with multidisciplinarity. In this sense, the biology and english teacher agreed that their subjects were the only ones in which they felt able to advise the teams, or at least to debate and counteract what the students were defending. These results link with the importance given in [**1**] to "metareflection questions about interdisciplinarity", meaning the reflection on the role and interactions of different disciplines and how to manage them in class. This reflection is crucial to planning the proposal, but also would have been here necessary to help teachers reflect on the epistemological and didactic nature of interdisciplinarity and on the strong constraints they were directly challenging.

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