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Erratum to: “Generalized Nadler G -contraction in cone metric spaces over Banach algebras endowed with a graph”
[Riv. Mat. Univ. Parma 6 (2015), 331–343]

In the proof of Theorem 3.2 of our paper [1] it has been assumed that $c_n^i = a^{2n}$ for all $n \in \mathbb{N}$, $i = 1, 2, \dots, k$, where c_n^i is an interior point of the cone P such that $\rho(c_n^i) < 1$ for all $n \in \mathbb{N}$, $i = 1, 2, \dots, k$ and $a \in P$ is the contractive vector of the mapping T . We point out that, in general, the assumption “ $c_n^i = a^{2n}$ ” is not valid since c_n^i ’s are in the interior of P , while a is in P (may not belong to the interior of P). In this note, we give an example which shows that such an assumption may not hold, hence we give a correction to the definition of generalized Nadler G -contraction.

We first give an example which shows that for $a \in P$ the vector a^n may not be in the interior of cone P , even if $\rho(a) < 1$.

Example 1. Let $\mathfrak{B} = \mathbb{R}^2$ with norm defined by $\|(x_1, x_2)\| = |x_1| + |x_2|$ and the multiplication defined by

$$(x_1, x_2)(y_1, y_2) = (x_1y_1, x_1y_2 + x_2y_1)$$

for all $(x_1, x_2), (y_1, y_2) \in \mathbb{R}^2$. Then $\mathfrak{B}$ is a Banach algebra with unit $e = (1, 0)$. Let $P = \{(x_1, x_2) \in \mathfrak{B} : x_1, x_2 \geq 0\}$. Then, P is a solid cone in $\mathfrak{B}$. Suppose, $a = (1/2, 0)$, then $a \in P$ and

$$\rho(a) = \lim_{n \rightarrow \infty} \|a^n\|^{1/n} = \lim_{n \rightarrow \infty} \|(1/2^n, 0)\|^{1/n} = \lim_{n \rightarrow \infty} \frac{1}{2} = \frac{1}{2} < 1.$$

Note that, the vector $a^n = (1/2^n, 0)$ belongs to P , but not to interior of P . Indeed, it is on the boundary of P .

The above example verifies the fact that we cannot assume $c_n^i = a^{2n}$ (since c_n^i must be in the interior of P). Note that, for any $a \in P^\circ$ (the interior of P) we have $a^n \in P^\circ$ for all $n \in \mathbb{N}$. Therefore, for validity of the proof of Theorem 3.2 of [1], the Definition 3.1 of [1] must be read as follows:

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Definition 1. Let (X, d) be a cone metric space over a Banach algebra $\mathfrak{B}$ and let $\mathcal{A}$ be a nonempty collection of nonempty subsets of X . A set-valued mapping $T: X \rightarrow \mathcal{A}$ is said to be a generalized Nadler G -contraction with contractive vector a , if for all $x, y \in X$ with $(x, y) \in E_G$:

- (i) there exists $a \in P^\circ$ such that $\rho(a) < 1$ and $H(Tx, Ty) \preceq ad(x, y)$;
- (ii) if $u \in Tx, v \in Ty$ are such that $d(u, v) \preceq d(x, y)$, then $(u, v) \in E_G$.

Similarly, the correct version of Definition 3.5 of [1] is given by:

Definition 2. Let $(X, \sqsubseteq, d)$ be an ordered complete cone metric space over a Banach algebra $\mathfrak{B}$ and let $\mathcal{A}$ be a nonempty collection of nonempty subsets of X . A set-valued mapping $T: X \rightarrow \mathcal{A}$ is said to be an ordered generalized Nadler contraction with contractive vector a , if for all $x, y \in X$ with $x \sqsubseteq y$:

- (i) there exists $a \in P^\circ$ such that $\rho(a) < 1$ and $H(Tx, Ty) \preceq ad(x, y)$;
- (ii) if $u \in Tx, v \in Ty$ are such that, $d(u, v) \preceq d(x, y)$, then $u \sqsubseteq v$.

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References

- [1] S. SHUKLA, *Generalized Nadler G -contraction in cone metric spaces over Banach algebras endowed with a graph*, Riv. Mat. Univ. Parma. **6** (2015), 331–343.

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