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On a non-existence result involving the fractional p-Laplacian

Abstract. We consider a nonlocal problem involving the fractional p-Laplacian operator in bounded smooth domains. A non-existence result is obtained via a comparison process. This result extends those done for the fractional Laplacian.

Keywords. Non-existence, weak solution, fractional p-Laplacian.

Mathematics Subject Classification (2010): 35R11.

1 - Introduction

Let $N \geq 1$ be an integer and let Ω be a bounded domain of $\mathbb{R}^N$. Consider the following nonlinear problem

$$(E_p^s) \qquad \begin{cases} (-\Delta)_p^s u = f(x,u) & \text{in } \Omega, \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega. \end{cases}$$

The nonlinear operator $(-\Delta)_p^s$ with $p > 1$ and $s \in (0,1)$ is the fractional p-Laplacian operator which is defined for any $x \in \mathbb{R}^N$ by

$$(1) \qquad (-\Delta_p)^s u(x) = 2 \lim_{\varepsilon \searrow 0} \int_{\mathbb{R}^N \setminus B_\varepsilon(x)} \frac{|u(x)-u(y)|^{p-2}}{|x-y|^{N+sp}} (u(x)-u(y))dy.$$

where $B_\varepsilon(x)$ denotes the open ball of radius $\varepsilon > 0$ and center x. System like (E_p^s) can be met in the field of game theory; see [**2**] for a complete framework of the subject. When $s = 1$, the fractional p-Laplacian becomes the well-known p-Laplacian op-

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erator. When $p = 2$, $(-\Delta)_p^s$ is reduced to be the linear operator $(-\Delta)^s$ usually called the fractional Laplacian and we obtain a semilinear problem which has been widely studied mainly to show non-existence results for nonlinear elliptic problems. In [**15**], X. Ros-Oton and J. Serra proved that (E_p^s) admits no positive and bounded solution if f is such that

$$\frac{N-2s}{2N} uf(u) \geq \int_0^u f(t)dt, \quad \forall u \in \mathbb{R}. \tag{2}$$

This result is obtained by using a version of the Pohozaev identity for the fractional Laplace problem. Indeed, for $s = 1$, $p = 2$, one retrieves the well-known non-existence condition like (2) established by S. I. Pohozaev in his pionner paper [**14**]. So, X. Ros-Oton and J. Serra generalized the results of [**14**] giving the fractional version of this identity. As an application of their work, they stated non-existence results for problem (E_p^s) with supercritical nonlinearities f in star-shaped domains Ω. The results of [**15**] are developed in a full paper [**18**]. More recently, these authors extended their results showing in [**17**] the non-existence of nontrivial bounded solutions to some nonlinear problems in the form

$$\begin{cases} Lu &= f(u) \quad \text{in } \Omega, \\ u &= 0 \quad \text{in } \mathbb{R}^N \setminus \Omega, \end{cases} \tag{3}$$

where L denotes a nonlocal operator in star-shaped domains Ω. An operator is nonlocal in the sense that one needs the value of a function in all Ω and not only in a neighbourhood of a point, to determine the effect of the operator on it. X. Cabré and Y. Sire interested in [**1**], in the existence, variational properties and asymptotic behaviour of particular solutions to (E_p^s) with $p = 2$. Nonlinear eigenvalues problems have been analyzed by A. Iannizzotto et al. in [**10**], showing the existence of eigenvalues. They established the existence of non-trivial weak solutions to (E_p^s) with $f = \lambda |u|^{p-2}u$, so demonstrating the existence of fractional eigenvalues λ. Then using Morse theory, they proved the existence of non-zero solutions in the p-superlinear case f. M. D'Elia and M. Gunzburger studied in [**3**] discretization methods for the nonlocal operator L, the fractional Laplacian operator being a special case of L. The main contribution of their paper is the demonstration of the convergence of the nonlocal operator L to the fractional Laplacian $-(\Delta)^s$ on bounded domains, under certain conditions. In [**9**], the eigenvalues of nonlocal operators, of which the fractional p-Laplacian operator is a particular case, were exploiting considering the weak solutions u to the nonlocal problem. Paper [**16**] is devoted to the Pohozaev identity for the fractional Laplacian $(-\Delta)^s$ with $s > 1$. This paper extends the results of [**18**] and give as application, a continuation property for the fractional eigenfunctions. We can also mention [**11**] in which R. Ignat presents some Pohozaev identities for an elliptic equation.

When $p > 1$, the study of existence of solutions to fractional p-Laplacian problems with weight was the object of [**12**]. Indeed, this paper is concerned with the existence of solutions to

$$\begin{cases} (-\Delta)_p^s u &= \varphi(x) f(u) \quad \text{in } \mathbb{R}^N, \\ u &\geq 0, \qquad\qquad u \neq 0. \end{cases}$$

In [**6**] A. Di Castro et al. investigated regularity results for nonlocal problem (3) with non-homogeneous Dirichlet condition, which can be reduced to problem (E_p^s) in some cases. R. Ferreira and M. Pérez-Llanos went further with limit problems for fractional p-Laplacian. They described in [**8**] the behaviour of solutions to (E_p^s) , as $p \to \infty$. A crucial paper for the theory of fractional Sobolev spaces $W^{s,p}$, $s \in (0,1)$, is [**7**], where the authors look the role of these spaces in the trace theory.

The present work aims to prove non-existence of nontrivial solutions for nonlinear problem (E_p^s) in the case $p > 1$ and $s \in (0,1)$.

Let Ω be a regular bounded open set of $\mathbb{R}^N$. Recall that

$$C^{0,\alpha}(\overline{\Omega}) = \left\{ u \in C(\overline{\Omega});\ sup_{\substack{x,y\in\Omega \\ x\neq y}} \frac{|u(x)-u(y)|}{|x-y|^{\alpha}} < \infty \right\},$$

$$C^{m,\alpha}(\overline{\Omega}) = \{u \in C^m(\overline{\Omega}), D^{\beta}u \in C^{0,\alpha}(\overline{\Omega}), \forall \beta \text{ with } |\beta| = m\},$$

where $D^{\beta}u = \dfrac{\partial^{|\beta|}u}{\partial x_1^{\beta_1}\dots\partial x_N^{\beta_N}} = \dfrac{\partial^{\beta_1+\dots+\beta_N}u}{\partial x_1^{\beta_1}\dots\partial x_N^{\beta_N}}$.

Let β be a strictly positive real, k is the greater integer such that $k < \beta$. $[\,\cdot\,]_{C^{k,\beta-k}(\Omega)}$ designates the seminorm on the space $C^{k,\beta-k}(\Omega)$,

$$[u]_{C^{k,\beta-k}(\Omega)} = \sup_{x,y\in\Omega,\, x\neq y} \frac{|D^k u(x) - D^k u(y)|}{|x-y|^{\beta-k}}.$$

The main result of this paper is as follows.

Theorem 1.1. *Consider Ω an open $C^{1,1}$ bounded set of $\mathbb{R}^N$. $\delta(x) = \operatorname{dist}(x, \partial\Omega)$. Let f be a locally Lipschitz function satisfying to*

$$\frac{N-sp}{Np} f(u)u \geq \int_0^u f(t)dt, \text{ for all } u \in \mathbb{R}. \tag{4}$$

Assume that u is a $W^{s,p}(\mathbb{R}^N)$ function which vanishes in $\mathbb{R}^N \setminus \Omega$, and such that u is of class $C^{0,1}(\Omega)$ and

$$[u]_{C^{0,1}(\{x\in\Omega;\delta(x)\geq\rho\})} \leq C\rho^{s-1}, \text{ for all } \rho \in (0,1),$$

then problem

$$\tag{5} \begin{cases} (-\Delta)^s_p u &= f(u) \quad \text{in } \Omega, \\ u &= 0 \quad \text{in } \mathbb{R}^N \setminus \Omega \end{cases}$$

admits no positive bounded solution. Moreover, if the inequality in (4) *is strict, then problem* (5) *admits no nontrivial bounded solution.*

Corollary 1.1. *Assume that the hypotheses of Theorem* 1.1 *hold. If* $\alpha \geq \dfrac{N(p-1)+sp}{N-sp}$, *then problem*

$$\tag{6} \begin{cases} (-\Delta)^s_p u &= u|u|^{\alpha-1} \quad \text{in } \Omega, \\ u &= 0 \quad \text{in } \mathbb{R}^N \setminus \Omega \end{cases}$$

admits no positive bounded solution. Moreover, if $\alpha > \dfrac{N(p-1)+sp}{N-sp}$, *then problem* (6) *admits no nontrivial bounded solution.*

The paper is organized as follows. In Section 2, we give the weak formulation to problem (E^s_p). Section 3 is devoted to the proof of the main result.

2 - Weak formulation

Let Ω be an open set of $\mathbb{R}^N$, star-shaped with respect to the origin of $\mathbb{R}^N$ and let $W^{s,p}(\mathbb{R}^N)$ be the fractional Sobolev space defined by

$$W^{s,p}(\mathbb{R}^N) = \left\{ u \in L^p(\mathbb{R}^N) : u \text{ measurable}, \frac{|u(x)-u(y)|}{|x-y|^{\frac{N}{p}+s}} \in L^p(\mathbb{R}^{2N}) \right\}$$

endowed with the natural norm

$$\|u\|_{W^{s,p}(\mathbb{R}^N)} = \left(\int_{\mathbb{R}^N} |u|^p dx + \int_{\mathbb{R}^{2N}} \frac{|u(x)-u(y)|^p}{|x-y|^{N+sp}} dxdy \right)^{\frac{1}{p}}.$$

Consider the Gagliardo (semi-)norm of all measurable function $u : \mathbb{R}^N \to \mathbb{R}$:

$$[u]_{W^{s,p}(\mathbb{R}^N)} = \left(\int\limits_{\mathbb{R}^{2N}} \frac{|u(x) - u(y)|^p}{|x - y|^{N+sp}} dxdy \right)^{\frac{1}{p}}.$$

See [7] for more details on the above notations. Now following [10], define the set

$$X(\Omega) = \{u \in W^{s,p}(\mathbb{R}^N) : u(x) = 0 \text{ a.e. in } \mathbb{R}^N \setminus \Omega\},$$

which can be renormed by setting $\|.\| = [.]_{W^{s,p}(\mathbb{R}^N)}$. The dual space of $(X(\Omega), \|.\|)$ is denoted by $(X(\Omega)^*, \|.\|_*)$. Define for all $u, v \in X(\Omega)$, the nonlinear operator $\mathcal{A} : X(\Omega) \to X(\Omega)^*$, by:

$$\langle \mathcal{A}(u), v \rangle = \int\limits_{\mathbb{R}^N} \int\limits_{\mathbb{R}^N} \frac{|u(x) - u(y)|^{p-2}(u(x) - u(y))(v(x) - v(y))}{|x - y|^{N+sp}} dxdy \tag{7}$$

where here and in the rest of the paper, the notation $\langle \cdot, \cdot \rangle$ designates the duality brackets between the spaces $X(\Omega)^*$ and $X(\Omega)$.

If u is smooth enough, this definition coincides with that of the fractional p-Laplacian (1). A (weak) solution of problem (E_p^s) is a function $u \in X(\Omega)$ such that for any $v \in X(\Omega)$,

$$\langle \mathcal{A}(u), v \rangle = \int\limits_{\Omega} f(x, u) v dx.$$

3 - Proof of the main result

Let u be a function in $X(\Omega)$. Following [18], define in $\mathbb{R}^N$ the function:

$$u_\lambda(x) = u(\lambda x).$$

Since $u :\equiv 0$ in $\mathbb{R}^N \setminus \Omega$, Ω is star-shaped then for $\lambda > 1$, it follows that $u_\lambda :\equiv 0$ in $\mathbb{R}^N \setminus \Omega$. It follows that $\int\limits_{\mathbb{R}^N} (-\Delta_p)^s u(x) u_\lambda(x) dx = \int\limits_{\Omega} (-\Delta_p)^s u(x) u_\lambda(x) dx$ and so,

$$\frac{d}{d\lambda}\Big|_{\lambda=1^+} \int\limits_{\mathbb{R}^N} u_\lambda(x)(-\Delta_p)^s u(x) dx = \frac{d}{d\lambda}\Big|_{\lambda=1^+} \int\limits_{\Omega} u_\lambda(x)(-\Delta_p)^s u(x) dx$$

where $\frac{d}{d\lambda}\Big|_{\lambda=1^+}$ is the derivative at $\lambda = 1$. We set $g(x) = (-\Delta_p)^s u(x)$ and taking account to the assumptions on u and f, we refer us to the proof of Proposition 1.6, page

10 in [**18**] and so, we argue similarly. Hence, the following identity holds

$$\int_{\Omega} (x \cdot \nabla u)\mathcal{A}(u)dx = \frac{d}{d\lambda}\Big|_{\lambda=1^+} \int_{\Omega} (-\Delta_p)^s u(x) u_\lambda(x)dx.$$

Then, we deduce

$$\int_{\Omega} (x \cdot \nabla u)\mathcal{A}(u)dx = \frac{d}{d\lambda}\Big|_{\lambda=1^+} \int_{\mathbb{R}^N} (-\Delta_p)^s u(x) u_\lambda(x)dx.$$

Consequently, using (7) with $v = u_\lambda$, and making the changes of variables $z = \lambda^{\frac{1}{p}} x$ and $\omega = \lambda^{\frac{1}{p}} y$, we obtain

$$\int_{\mathbb{R}^N} (-\Delta_p)^s u(x) u_\lambda(x)dx = \int_{\mathbb{R}^N}\int_{\mathbb{R}^N} \frac{|u(x)-u(y)|^{p-2}(u(x)-u(y))}{|x-y|^{N+sp}}(u(\lambda x)-u(\lambda y))dxdy.$$

More precisely, we have

$$\begin{aligned}
\langle \mathcal{A}(u), u_\lambda\rangle = &\int_{\mathbb{R}^{2N}} \frac{|u(\lambda^{-\frac{1}{p}}z)-u(\lambda^{-\frac{1}{p}}\omega)|^{p-2}}{|\lambda^{-\frac{1}{p}}z-\lambda^{-\frac{1}{p}}\omega|^{N+sp}}(u(\lambda^{-\frac{1}{p}}z)-u(\lambda^{-\frac{1}{p}}\omega))\\
&\times (u(\lambda^{\frac{p-1}{p}}z)-u(\lambda^{\frac{p-1}{p}}\omega))\lambda^{\frac{-2N}{p}}dzd\omega\\
= \lambda^{\frac{sp-N}{p}} &\int_{\mathbb{R}^{2N}} \frac{|u(\lambda^{-\frac{1}{p}}z)-u(\lambda^{-\frac{1}{p}}\omega)|^{p-2}}{|z-\omega|^{N+sp}}(u(\lambda^{-\frac{1}{p}}z)-u(\lambda^{-\frac{1}{p}}\omega))\\
&\times (u(\lambda^{\frac{p-1}{p}}z)-u(\lambda^{\frac{p-1}{p}}\omega))dzd\omega\\
= \lambda^{\frac{sp-N}{p}} &\int_{\mathbb{R}^{2N}} \left(\frac{|u(\lambda^{-\frac{1}{p}}z)-u(\lambda^{-\frac{1}{p}}\omega)|}{|\lambda^{-\frac{1}{p}}z-\lambda^{-\frac{1}{p}}\omega|^{\frac{N+sp}{p}}}\right)^{p-2}\frac{u(\lambda^{-\frac{1}{p}}z)-u(\lambda^{-\frac{1}{p}}\omega)}{|\lambda^{-\frac{1}{p}}z-\lambda^{-\frac{1}{p}}\omega|^{\frac{N+sp}{p}}}\\
&\times \frac{u(\lambda^{\frac{p-1}{p}}z)-u(\lambda^{\frac{p-1}{p}}\omega)}{|\lambda^{\frac{p-1}{p}}z-\lambda^{\frac{p-1}{p}}w|^{\frac{N+sp}{p}}}dzdw
\end{aligned}$$

because we have remarked that $\lambda^{-\frac{p-2}{p}\frac{N+sp}{p}}\lambda^{-\frac{1}{p}\frac{N+sp}{p}}\lambda^{\frac{p-1}{p}\frac{N+sp}{p}} = 1$ and $|z-\omega|^{\frac{N+sp}{p}(p-2)+\frac{N+sp}{p}+\frac{N+sp}{p}} = |z-\omega|^{N+sp}$.

Thus,

$$(8)\qquad \int_{\Omega} (x\cdot\nabla u)A(u)dx = \frac{sp-N}{p}\int_{R^{2N}} \left(\frac{|u(z)-u(\omega)|}{|z-\omega|^{\frac{N+sp}{p}}}\right)^{p-2}\frac{u(z)-u(\omega)}{|z-\omega|^{\frac{N+sp}{p}}} \times \frac{u(z)-u(\omega)}{|z-w|^{\frac{N+sp}{p}}}dzdw + \frac{d}{d\lambda}\Big|_{\lambda=1^+} I_\lambda,$$

where

$$I_\lambda = \int\limits_{R^{2N}} \left(\frac{|u(\lambda^{-\frac{1}{p}}z) - u(\lambda^{-\frac{1}{p}}\omega)|}{|\lambda^{-\frac{1}{p}}z - \lambda^{-\frac{1}{p}}\omega|^{\frac{N+sp}{p}}} \right)^{p-2} \frac{u(\lambda^{-\frac{1}{p}}z) - u(\lambda^{-\frac{1}{p}}\omega)}{|\lambda^{-\frac{1}{p}}z - \lambda^{-\frac{1}{p}}\omega|^{\frac{N+sp}{p}}} \times \frac{u(\lambda^{\frac{p-1}{p}}z) - u(\lambda^{\frac{p-1}{p}}\omega)}{|\lambda^{\frac{p-1}{p}}z - \lambda^{\frac{p-1}{p}}w|^{\frac{N+sp}{p}}} dzdw.$$

Then we can write

$$\int\limits_{\Omega} (x \cdot \nabla u)\mathcal{A}(u)dx - \frac{sp-N}{p} \int\limits_{\mathbb{R}^{2N}} \frac{|u(x)-u(y)|^p}{|x-y|^{N+sp}} dxdy = \frac{d}{d\lambda}\Big|_{\lambda=1^+} I_\lambda. \tag{9}$$

We have:

$$\frac{d}{d\lambda}\Big|_{\lambda=1^+} I_\lambda = \lim_{\lambda\to 1^+} \frac{I_\lambda - I_1}{\lambda - 1},$$

where $I_1 = \int\limits_{\mathbb{R}^{2N}} \frac{|u(x)-u(y)|^p}{|x-y|^{N+sp}} dxdy$. Let us study the sign of $\frac{d}{d\lambda}\Big|_{\lambda=1^+} I_\lambda$. In view of the Cauchy-Schwarz inequality, we can write:

$$I_\lambda \le \int\limits_{\mathbb{R}^{2N}} \left(\frac{|u(\lambda^{-\frac{1}{p}}z) - u(\lambda^{-\frac{1}{p}}\omega)|}{|\lambda^{-\frac{1}{p}}z - \lambda^{-\frac{1}{p}}\omega|^{\frac{N+sp}{p}}} \right)^{p-1} \frac{|u(\lambda^{\frac{p-1}{p}}z) - u(\lambda^{\frac{p-1}{p}}\omega)|}{|\lambda^{\frac{p-1}{p}}z - \lambda^{\frac{p-1}{p}}w|^{\frac{N+sp}{p}}} dzdw.$$

Set $\phi_\lambda(z,\omega) = \frac{u(\lambda^{-\frac{1}{p}}z) - u(\lambda^{-\frac{1}{p}}\omega)}{|\lambda^{-\frac{1}{p}}z - \lambda^{-\frac{1}{p}}\omega|^{\frac{N+sp}{p}}}$, then

$$I_\lambda \le \int\limits_{\mathbb{R}^{2N}} |\phi_\lambda(z,\omega)|^{p-1} |\phi_\lambda(\lambda z, \lambda\omega)| dzd\omega.$$

But applying Hölder's inequality for each $\lambda > 1$ we get:

$$I_\lambda \le \left(\int\limits_{\mathbb{R}^{2N}} |\phi_\lambda(z,\omega)|^p dzd\omega \right)^{\frac{p-1}{p}} \left(\int\limits_{\mathbb{R}^{2N}} |\phi_\lambda(\lambda z, \lambda\omega)|^p dzd\omega \right)^{\frac{1}{p}} \tag{10}$$

since $\phi_\lambda(.,.) \in L^p$ (then $|\phi_\lambda(.,.)|^{p-1} \in L^{\frac{p}{p-1}}$) and $\phi_\lambda(\lambda(.,.)) \in L^p$. On the one hand, making the changes of variables $x = \lambda^{-\frac{1}{p}}z$ and $y = \lambda^{-\frac{1}{p}}\omega$, it follows that:

$$\begin{aligned} \left(\int\limits_{\mathbb{R}^{2N}} |\phi_\lambda(z,\omega)|^p dzd\omega \right)^{\frac{p-1}{p}} &= \left(\lambda^{\frac{2N}{p}} \int\limits_{\mathbb{R}^{2N}} \frac{|u(x)-u(y)|^p}{|x-y|^{N+sp}} dxdy \right)^{\frac{p-1}{p}} \\ &= \lambda^{\frac{2N}{p}\frac{p-1}{p}} \left(\int\limits_{\mathbb{R}^{2N}} \frac{|u(x)-u(y)|^p}{|x-y|^{N+sp}} dxdy \right)^{\frac{p-1}{p}}. \end{aligned}$$

On the other hand, making the changes of variables $X = \lambda^{\frac{p-1}{p}} z$ and $Y = \lambda^{\frac{p-1}{p}} \omega$, we get:

$$\left(\int_{\mathbb{R}^{2N}} |\phi_\lambda(\lambda z, \lambda\omega)|^p dzd\omega \right)^{\frac{1}{p}} = \left(\lambda^{-2N\frac{p-1}{p}} \int_{\mathbb{R}^{2N}} \frac{|u(X) - u(Y)|^p}{|X - Y|^{N+sp}} dXdY \right)^{\frac{1}{p}}$$

$$= \lambda^{-\frac{2N}{p}\frac{p-1}{p}} \left(\int_{\mathbb{R}^{2N}} \frac{|u(X) - u(Y)|^p}{|X - Y|^{N+sp}} dXdY \right)^{\frac{1}{p}}.$$

Coming back to (10), we deduce that:

$$I_\lambda \leq \int_{\mathbb{R}^{2N}} \frac{|u(x) - u(y)|^p}{|x - y|^{N+sp}} dxdy = I_1.$$

Consequently, the quantity $\dfrac{I_\lambda - I_1}{\lambda - 1}$ is negative since $\lambda > 1$ and then, $\dfrac{d}{d\lambda}\Big|_{\lambda=1^+} I_\lambda$ is negative too. Since u is a solution of (5), the identity holds

$$\int_\Omega (x \cdot \nabla u)\mathcal{A}(u)dx = \int_\Omega (x \cdot \nabla u)f(u)dx = -N \int_\Omega F(u)dx \tag{11}$$

where $F(t) = \int_0^t f(\xi)d\xi$. And finally, it follows from (9) that:

$$-N \int_\Omega F(u)dx - \frac{sp - N}{p} \int_\Omega f(u)udx \leq 0,$$

from which we conclude that:

$$\int_\Omega N \left(\frac{N - sp}{Np} f(u)u - \int_0^u f(\xi)d\xi \right) dx \leq 0. \tag{12}$$

Now setting $f(u) = u|u|^{\alpha-1}$ in (12) yields

$$\int_\Omega \left(\frac{N - sp}{Np} - \frac{1}{\alpha + 1} \right) |u|^{\alpha+1} dx \leq 0,$$

which completes the proof of Theorem 1.1 □

Now consider that $f(x,u)$ instead of $f(u)$. The following result holds

Corollary 3.1. *Consider Ω an open bounded set of $\mathbb{R}^N$ of class $C^{1,1}$. Let f a function of class $C^{0,1}_{\text{loc}}(\overline{\Omega}\times\mathbb{R})$. Assume that u is a $W^{s,p}(\mathbb{R}^N)$ function which vanishes in $\mathbb{R}^N\setminus\Omega$ and u of class $C^{0,1}(\Omega)$.*

Then if the domain Ω is star-shaped, and the condition holds

$$\frac{N-sp}{p}f(x,t)u \geq NF(x,t) + x\cdot F_x(x,t), \ \text{for all } x\in\Omega \text{ and } t\in\mathbb{R}, \tag{13}$$

then problem (E^s_p) admits no positive bounded solution. Moreover, if the inequality (13) *is strict, then problem (E^s_p) admits no nontrivial bounded solution.*

Proof. Considering $f(x,u)$ instead of $f(u)$, then (11) can be rewritten in the form:

$$\int_\Omega (x\cdot\nabla u)\mathcal{A}(u)dx = \int_\Omega (x\cdot\nabla u)f(x,u)dx = -N\int_\Omega F(x,u)dx - \int_\Omega x\cdot F_x(x,u)dx.$$

Consequently (9) is equivalent to

$$N\int_\Omega F(x,u)dx + \int_\Omega x\cdot F_x(x,u)dx + \frac{sp-N}{p}f(x,u)udx \geq 0,$$

where $F(x,u) = \int_0^u f(x,\xi)d\xi.$ □

Our study shows that the well-known classical non-existence results (see for instance [**4, 5, 13**]) can be interpreted as a limiting case of the diffusion fractional. Moreover, they extend for a non-linear fractional system those obtained by X. Ros-Oton and J. Serra in [**18**] for a linear fractional operator. However, the tools used in [**18**] seem fail if we consider the nonlinear case in perspective to the establishing of a typical Pohozaev identity taking account the nonlinearity of the fractional p-Laplacian operator. This open question is actually investigated by the authors and it will be presented in a next paper. An another open problem is the following: Is it possible to obtain without regularity assumption a non-existence result of any solution belonging in $X(\Omega)\bigcap L^\infty(\Omega)$?

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