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C. L. RISHISHWAR (*)

On the ψ-type, lower ψ-type and ψ-λ-type of the functions represented by the series $\sum_{n=0}^{\infty} a_n \exp(\lambda_n \psi(z))$. (**)

1. - Introduction.

So far the growth of entire functions with Taylor series expansion and that of functions represented by Dirichlet series, have been studied separately by the different workers in the two fields. With an object to unify the two approaches, a real series was studied in recent papers $[1]_2$, $[1]_3$, $[1]_4$, $[1]_5$, $[1]_6$. To be more precise, here in this paper, we examine the series ($[1]_1$)

$$\sum_{n=0}^{\infty} a_n \cdot \exp(\lambda_n \cdot \psi(z)) \tag{1.1}$$

where:

$$\limsup_{n\to\infty} n/\lambda_n = D < \infty\,, \qquad 0 = \lambda_0 < \lambda_1 < \dots (\lambda_n \xrightarrow[n\to\infty]{} \infty); \tag{1.2}$$

$\{a_n\}$ $(n = 0, 1, 2, \dots)$ is a sequence of complex numbers; $\psi(z)$ is an analytic function of complex variable z, analytic in the region $\log|\exp\psi(z)| \leqslant R$ and satisfying the following conditions:

(1.3)
(a) $\psi(z)$ has an inverse, that is, if $y = \psi(z)$, then there exists a function ψ^{-1} such that $\psi^{-1}(y) = z$,

(b) $\psi(z) = \log|\exp\psi(z)| + ic$

where c is a real function depending on z and varying in the interval $-(\pi \text{ or } \infty) \leqslant c \leqslant +(\pi \text{ or } \infty)$.

(*) Indirizzo: Gur Narain Khattri College, Civil Lines, Kanpur-1, India.
(**) Ricevuto: 1-IX-1971.

Let R_c and R_a be the « extent of ψ-convergence » and « extent of absolute ψ-convergence » respectively of the series in (1.1). If $R_c = \infty$, $R_a = \infty$, then the series in (1.1) is convergent for all values of $R < R_c$, and the sum function $F(z)$, represented by the series in (1.1), is defined and analytic in the region $\log |\exp(\psi z)| \leqslant R \leqslant R_c$:

Remark. It is interesting that, if we take $\psi(z) = \log z$ and $\lambda_n = n$, or $\psi(s) = s$, the series in (1.1) assumes the form of a Taylor series or a Dirichlet series respectively. Also, $\log |\exp \psi(z)| \leqslant R$ defines a circular region with its centre at the origin and radius $\exp(R)$ as well as a half plane $\operatorname{Re} s = \sigma \leqslant R$.

Now, let

$$F(z) = \sum_{n=0}^{\infty} a_n \cdot \exp[\lambda_n \cdot \psi(z)]$$

be a function, satisfying the conditions mentioned in (1.2) and (1.3). Again, let maximum term $\mu(R, F) = \sup[|a_n| \cdot |\exp \psi(z)|^{\lambda_n}]$ $(n \leqslant 0)$; rank of the maximum term $\nu(R, F) = \max[n;\ |a_n| \cdot |\exp \psi(z)|^{\lambda_n} = \mu(R, F)]$. Then $\mu(R, F) = |a_{\nu(R,F)}|\, |\exp \psi(z)|\lambda_{\nu(R,F)}$. Further, let

$$M(R) = \underset{-(\pi \text{ or } \infty) \leqslant c \leqslant +(\pi \text{ or } \infty)}{\text{maximum}} |F(z)| \quad \text{for} \quad \log |\exp \psi(z)| \leqslant R < R_c\,.$$

Beginning with the aim to study the rate of growth of the function $F(z)$ represented by the series in (1.1), we define the measure of rate of growth of the function $F(z)$. Let

$$\lim_{R\to\infty} \begin{matrix}\sup\\ \inf\end{matrix} \frac{\log\log M(R)}{\log |\exp \psi(z)|} = \begin{matrix}\varrho\\ \lambda\end{matrix} \qquad (0 \leqslant \lambda \leqslant \varrho \leqslant \infty). \tag{1.4}$$

We shall refer to the constants ϱ and λ, as defined in (1.4), as the ψ-order and the lower ψ- order respectively of the function $F(z)$, which shall be said of regular ψ-growth when $\varrho = \lambda$. The justification for this, lies in the fact that ϱ and λ depend on the function $\psi(z)$.

A better estimate of the growth of the function $F(z)$ in relation to the function $\psi(z)$ is obtained, if we consider the limit of $\log M(R)/|\exp \psi(z)|^{\varrho}$. Thus, let

$$\lim_{R\to\infty} \begin{matrix}\sup\\ \inf\end{matrix} \frac{\log M(R)}{|\exp \psi(z)|^{\varrho}} = \begin{matrix}T\\ t\end{matrix} \qquad (0 \leqslant t \leqslant T < \infty)\,, \tag{1.5}$$

where ϱ $(0 < \varrho < \infty)$ is the ψ-order of the function $F(z)$. We define « T » to be the ψ-type and «t» the lower ψ-type of the function $F(z)$ of ψ-order ϱ, and in case the limit in (1.5) exists i.e. $T = t$ ($< \infty$) and $\varrho = \lambda$, we say that «the function $F(z)$ is of perfectly regular ψ-growth».

Here, in this paper, we obtain expressions for ψ-type and lower ψ-type in terms of the coefficients of the terms of the series in (1.1). Another concept of ψ-λ-type is given here when lower ψ-type is zero. In last section of this paper, we define ψ-growth numbers and obtain a number of inequalities connecting them with ψ-type and ψ-order, for which we shall prove some lemmas in the beginning.

2. – Lemma 1. *Let* $F(z) = \sum_{n=0}^{\infty} a_n \cdot \exp(\lambda_n \cdot \psi(z))$ *be a function, analytic in the region* $\log|\exp \psi(z)| \leqslant R$, *then for every* z *in this region*

$$(2.1) \qquad a_n = \lim_{C \to (\pi \text{ or } \infty)} \frac{1}{2Ci} \int_{\psi^{-1}(R-Ci)}^{\psi^{-1}(R+Ci)} F(z) \cdot \exp[-\lambda_n \cdot \psi(z)] \cdot \psi'(z)\, dz,$$

where $\psi(z) = \log|\exp \psi(z)| + ci$ *and integration being taken along the path* $\log|\exp \psi(z)| = R$, *also when the limit* $C \to \pi$, *is considered,* $\lambda_n s$ *are taken to be integers.*

Proof. We have

$$F(z) \cdot \exp(-\lambda_n \cdot \psi(z)) = a_0 \exp([\lambda_0 - \lambda_n] \cdot \psi(z)) + \dots +$$
$$+ a_{n-1} \cdot \exp([\lambda_{n-1} - \lambda_n] \psi(z)) + a_n + a_{n+1} \cdot \exp([\lambda_{n+1} - \lambda_n] \cdot \psi(z)) + \dots .$$

It can easily be shown that the series is uniformly convergent in the region $\log|\exp \psi(z)| = R$, hence integrating along the path $\log|\exp \psi(z)| = R$, we have

$$\int_{R-Ci}^{R+Ci} F(z) \exp[-\lambda_n \cdot \psi(z)]\, d\{\psi(z)\} = \int_{R-Ci}^{R+Ci} a_0 \exp[-\lambda_n \cdot \psi(z)]\, d\{\psi(z)\} + \dots +$$
$$+ \int_{R-Ci}^{R+Ci} a_{n-1} \exp([\lambda_{n-1} - \lambda_n] \cdot \psi(z))\, d\{\psi(z)\} + \int_{R-Ci}^{R+Ci} a_n\, d\{\psi(z)\} +$$
$$+ \int_{R-Ci}^{R+Ci} a_{n+1} \exp([\lambda_{n+1} - \lambda_n] \cdot \psi(z))\, d\{\psi(z)\} + \dots .$$

Therefore, from (1.3), we have

$$\frac{1}{2Ci}\int_{\psi^{-1}(R-Ci)}^{\psi^{-1}(R+Ci)} F(z)\cdot\exp(-\lambda_n\cdot\psi(z))\cdot\psi'(z)\,dz = \frac{1}{2C}\cdot a_0\exp(-\lambda_n\cdot R)\int_{-C}^{+C}\exp(-\lambda_n\cdot ci)\,dc + \ldots$$

$$\ldots + \frac{1}{2C}\cdot a_{n-1}\cdot\exp(R[\lambda_{n-1}-\lambda_n])\int_{-C}^{+C}\exp([\lambda_{n-1}-\lambda_n]\cdot ci)\,dc + a_n +$$

$$+\frac{1}{2C}\cdot a_{n+1}\cdot\exp(R[\lambda_{n+1}-\lambda_n])\cdot\int_{-C}^{+C}\exp([\lambda_{n+1}-\lambda_n]\cdot ci)\,dc + \ldots =$$

$$= \ldots + \frac{1}{2C}\cdot a_{n-1}\cdot\exp(R[\lambda_{n-1}-\lambda_n])\int_{-C}^{+C}[\cos\{\lambda_{n-1}-\lambda_n\}c + i\sin\{\lambda_{n-1}-\lambda_n\}c]\,dc +$$

$$+ a_n + \frac{1}{2C}\cdot a_{n+1}\cdot\exp(R[\lambda_{n+1}-\lambda_n])\int_{-C}^{+C}[\cos\{\lambda_{n+1}-\lambda_n\}c + i\sin\{\lambda_{n+1}-\lambda_n\}c]\,dc\,.$$

Proceeding to limits, we obtain

$$\lim_{C\to(\pi \text{ or } \infty)} \frac{1}{2Ci}\int_{\psi^{-1}(R-Ci)}^{\psi^{-1}(R+Ci)} F(z)\cdot\exp(-\lambda_n\cdot\psi(z))\cdot\psi'(z)\,dz = a_n\,,$$

since all the integrals in the right hand side of the equality vanish in view of the limits under consideration. While considering the limit $C\to\pi$, $\lambda_n s$ are taken to be integers.

Remark. The formlae for the coefficients a_n, in case of power series and Dirichlet series follow from the expression obtained here. The method used here is different from that of Cauchy and J. F. Ritt.

Lemma 2. *Let* $F(z)=\sum_{n=0}^{\infty} a_n\exp(\lambda_n\cdot\psi(z))$ *be a function analytic in the region* $\log|\exp\psi(z)|\leqslant R \quad (R<R_c)$, *then*

$$(2.2)\qquad |a_n|\cdot|\exp(\psi(z)|^{\lambda_n})\leqslant M(R) \qquad \textit{for all values of } n\,,$$

where $M(R)$ *is the upper bound of* $|F(z)|$ *on the path* $\log|\exp\psi'(z)| = R.$

Proof. From Lemma 1, we have

$$a_n = \lim_{C \to (\pi \text{ or } \infty)} \frac{1}{2Ci} \int_{\psi^{-1}(R-Ci)}^{\psi^{-1}(R+Ci)} F(z) \cdot \exp\left(-\lambda_n \cdot \psi(z)\right) \cdot \psi'(z)\, dz .$$

Therefore,

$$|a_n| \leqslant \frac{M(R)}{|\exp \psi(z)|^{\lambda_n}} \lim_{C \to (\pi \text{ or } \infty)} \frac{1}{2Ci} \int_{R-Ci}^{R+Ci} d\{\psi(z)\} ,$$

or

$$|a_n| \leqslant \frac{M(R)}{|\exp \psi(z)|^{\lambda_n}}$$

and hence the proof.

Theorem 1. *The necessary and sufficient condition that the function*

$$F(z) = \sum_{n=0}^{\infty} a_n \exp\left(\lambda_n \cdot \psi(z)\right)$$

to be of ψ-type T $(0 \leqslant T < \infty)$ of finite ψ-order $\varrho > 0$, is that

$$\limsup_{n \to \infty} (\lambda_n / e\varrho) \cdot |a_n|^{\varrho/\lambda_n} = T . \tag{2.3}$$

Proof. Let us set

$$\limsup_{n \to \infty} (\lambda_n / e\varrho) \cdot |a_n|^{\varrho/\lambda_n} = \nu .$$

Suppose that $0 \leqslant \nu < \infty$. Let $\varepsilon > 0$, we have

$$|a_n| < \left[\frac{(\nu + \varepsilon) \cdot e\varrho}{\lambda_n}\right]^{\lambda_n/\varrho} \qquad \text{for } n > n_0 .$$

Hence

$$|F(z)| \leqslant \sum_{n=0}^{n_0} |a_n| \cdot |\exp \psi(z)|^{\lambda_n} + \sum_{n_0+1}^{\infty} |a_n| \cdot |\exp \psi(z)|^{\lambda_n}$$

$$\leqslant A \cdot |\exp \psi(z)|^{\lambda_{n_0}} + \sum_{n_0+1}^{\infty} \left[\frac{(\nu + \varepsilon) e\varrho \cdot |\exp \psi(z)|^{\varrho}}{\lambda_n}\right]^{\lambda_n/\varrho} .$$

The general term on the right hand side does not exceed its maximum which is $\exp\left[(\nu+\varepsilon)\cdot|\exp\psi(z)|^{\varrho}\right]$ attained for $\lambda_n=(\nu+\varepsilon)\cdot\varrho|\exp\psi(z)|^{\varrho}$.

Now choose an integer M such that

$$\lambda_M\leqslant(\nu+2\varepsilon)e\varrho\cdot|\exp\psi(z)|^{\varrho}<\lambda_{M+1}\,.$$

When $\lambda_{n_0+1}\leqslant\lambda_n\leqslant\lambda_M$, we have

$$\sum_{n_0+1}^{M}\left[\frac{(\nu+\varepsilon)e\varrho\cdot|\exp\psi(z)|^{\varrho}}{\lambda_n}\right]^{\lambda_n/\varrho}\leqslant(\text{number of terms})\cdot\exp\left[(\nu+\varepsilon)\cdot|\exp\psi(z)|^{\varrho}\right]$$

$$=O\left[\exp\{(\nu+\varepsilon)\cdot|\exp\psi(z)|^{\varrho}\}\right].$$

Also

$$\sum_{M+1}^{\infty}\left[\frac{(\nu+\varepsilon)e\varrho\cdot|\exp(\psi(z))|^{\varrho}}{\lambda_n}\right]^{\lambda_n/\varrho}<\sum_{M+1}^{\infty}\left[\frac{\nu+\varepsilon}{\nu+2\varepsilon}\right]^{\lambda_n/\varrho}=O(1)\,.$$

Thus

$$T=\limsup_{R\to\infty}\frac{\log M(R)}{|\exp\psi(z)|^{\varrho}}\leqslant\nu\,. \tag{2.4}$$

Again, suppose $0<\nu<\infty$, we have for an infinity of n

$$|a_n|>\left[\frac{(\nu-\varepsilon)e\varrho}{\lambda_n}\right]^{\lambda_n/\varrho}\qquad(0<\varepsilon<v)\,.$$

If, for these values of λ_n, we take $|\exp\psi(z)|^{\varrho}=\lambda_n/(\nu-\varepsilon)\varrho$, then from Lemma 2, we have

$$M(R)\geqslant|a_n|\cdot|\exp\psi(z)|^{\lambda_n}>\left[\frac{(\nu-\varepsilon)e\varrho\cdot|\exp\psi(z)|^{\varrho}}{\lambda_n}\right]^{\lambda_n/\varrho}=$$

$$=\exp(\lambda_n/\varrho)=\exp\left[(\nu-\varepsilon)\cdot|\exp\psi(z)|^{\varrho}\right]$$

for a sequence of values of R tending to infinity. Thus

$$T\geqslant\nu\,. \tag{2.5}$$

Hence the result in (2.3) follows from (2.4) and (2.5).

Theorem 2. *Let* $F(z)=\sum_{n=0}^{\infty} a_n\cdot\exp(\lambda_n\cdot\psi(z))$ *be a function of* ψ*-order* ϱ $(0<\varrho<\infty)$ *such that*

$$\liminf_{R\to\infty}\frac{\log M(R)}{|\exp\psi(z)|^{\varrho}}=t\,.$$

If $\lambda_{n+1}\sim\lambda_n$, *then*

$$t\geqslant\liminf_{n\to\infty}\frac{\lambda_n}{e\varrho}\cdot|a_n|^{\varrho/\lambda_n}\,, \tag{2.6}$$

and further, if $\log|a_n/a_{n+1}|/(\lambda_{n+1}-\lambda_n)$ *forms a non-decreasing function of* n *for* $n>n_0$ *then*

$$t=\liminf_{n\to\infty}\frac{\lambda_n}{e\varrho}\cdot|a_n|^{\varrho/\lambda_n}\,. \tag{2.7}$$

The proof of the theorem is omitted as it is similar to that given for the real series ([**1**]$_2$, p. 87).

Theorem 3. *Let* $F(z)$, *represented by the series in* (1.1), *be a function of* ψ*-order* ϱ *and lower* ψ*-order* λ $(0\leqslant\lambda<\varrho<\infty)$. *Then*

$$\liminf_{R\to\infty}\frac{\log M(R)}{|\exp\psi(z)|^{\varrho}}=0\,,\qquad \liminf_{R\to\infty}\frac{\lambda_{\nu(R,F)}}{|\exp\psi(z)|^{\varrho}}=0\,, \tag{2.8}$$

i.e. *the lower* ψ*-type of the function* $F(z)$ *of irregular* ψ*-growth of finite* ψ*-order* ϱ, *is zero.*

Proof. From (1.4) we have

$$\log M(R)>|\exp\psi(z)|^{(\lambda-\varepsilon)}\qquad\text{for any } \varepsilon>0 \text{ and } R>R_0=R_0(\varepsilon)\,. \tag{2.9}$$

$$\log M(R)<|\exp\psi(z)|^{(\varrho+\varepsilon)}\qquad\text{for a sequence of values of } R\to\infty\,. \tag{2.10}$$

Dividing (2.9), (2.10) by $|\exp\psi(z)|^{\varrho}$ and then proceeding to limits, the argument shows that

$$\liminf_{R\to\infty}\frac{\log M(R)}{|\exp\psi(z)|^{\varrho}}=0\,.$$

Also, proceeding on the lines as above, we obtain the second part of the theorem.

3. – The fact that

$$\liminf_{R\to\infty} \frac{\mathrm{lo}\, Mg\ (R)}{|\exp\psi(z)|^{\varrho}} = 0 \qquad \text{when} \quad 0 \leqslant \lambda < \varrho < \infty\,,$$

opens the question of comparing the function $\log|F(z)|$ with the function $|\exp\psi(z)|^{\lambda}$ when $0<\lambda<\varrho<\infty$. Evidently, since $\lambda<\varrho$,

$$\limsup_{R\to\infty} \frac{\log M(R)}{|\exp\psi(z)|^{\lambda}} = \infty$$

yet $\liminf\limits_{R\to\infty} [\log M(R)]/|\exp\psi(z)|^{\lambda}$ may still be a finite constant. We shall refer to this constant as ψ-λ-type of the function $F(z)$ and denote it by t_{λ}.

A result, similar to that of Theorem 2, also holds for ψ-λ-type t_{λ}, which can be proved on the lines of that theorem.

4. – In this section, we define *ψ-growth number* ν and *lower ψ-growth number* δ as

$$\lim_{R\to\infty} \begin{matrix}\sup\\ \inf\end{matrix} \frac{\lambda_{\nu(R,F)}}{|\exp\psi(z)|^{\varrho}} = \begin{matrix}\gamma\\ \delta\end{matrix}\,, \qquad (0\leqslant\delta\leqslant\gamma<\infty). \tag{4.1}$$

Theorem 4. *If the symbols have the meanings as mentioned above, then*

$$\begin{cases} \text{(i)} & \gamma \geqslant \varrho T \geqslant \varrho t \geqslant \delta\,, \\ \text{(ii)} & \gamma \geqslant \varrho T \geqslant \dfrac{\gamma \exp(\delta/\gamma)}{e} \geqslant \delta\,, \\ \text{(iii)} & \gamma \geqslant \delta\left(1+\log\dfrac{\gamma}{\delta}\right) \geqslant \varrho t \geqslant \delta\,, \\ \text{(iv)} & \gamma + \delta \leqslant e\varrho T\,, \\ \text{(v)} & \textit{equality cannot hold simultaneously in (iv) and } \delta \leqslant \varrho T\,. \end{cases} \tag{4.2}$$

The proof of the theorem is omitted as it is based on the proof of a similar theorem for the real series in a paper ([**1**]$_2$, pp. 91-93).

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References.

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Abstract.

Consider the series (*) $\sum_{n=0}^{\infty} a_n e^{\lambda_n \cdot \psi(z)}$ *where* $\limsup n/\lambda_n = D < \infty$, $0 = \lambda_0 < \lambda_1 < \lambda_2 < < \lambda_3 < \ldots < \lambda_n \xrightarrow{n \to \infty} \infty$, $\{a_n\}$ $(n = 0, 1, 2, \ldots)$ *is a sequence of complex numbers and* $\psi(z)$ *is an analytic function of complex variable* z, *analytic in the region* $\log|\exp \psi(z)| \leqslant R$, *and satisfying the conditions*: (a) $\psi(z)$ *has an inverse*, (b) $\psi(z) = \log|\exp \psi(z)| - ic$, *where* c *is a real function depending on* z *and varying in the interval* $-(\pi$ *or* $\infty) \leqslant c \leqslant +(\pi$ *or* $\infty)$. *Let* R_c *and* R_a *be the «extent of ψ-convergence» and «extent of absoluta ψ-convergence» of the series mentioned above. If* $R_c = \infty$ *and* $R_a = \infty$, *then the series is convergent for all values of* $R < R_c$ *and the sum function* $F(z)$ *represented by this series is defined and analytic in the region* $\log|\exp \psi(z)| \leqslant R \leqslant R_c$. *Taking* $\psi(z) = \log z$ *and* $\lambda_n = n$ *or* $\psi(s) = s$ *the series assumes the form of a Taylor series or a Dirichlet series respectively. Also,* $\log|\exp \psi(z)| \leqslant R$ *defines a circular region with its centre at the origin and radius* e^R *as well as a half plane* $\text{Re}\, s = \sigma \leqslant R$, *thus the series unifies the two theories of functions represented by Taylor series and Dirichlet series respectively. In this paper, we study the rate of growth of the function represented by the above series. The measures of rate of growth are defined and expressions for them have been obtained.*

Lemma. *Let* $F(z) = \sum_{n=0}^{\infty} a_n \cdot \exp(\lambda_n \cdot \psi(z))$ *be a function, analytic in the region* $\log|\exp \psi(z)| \leqslant R$, *then for every* z *in this region*

$$a_n = \lim_{C \to (\pi \text{ or } \infty)} (1/2Ci) \int_{\psi^{-1}(R+Ci)}^{\psi^{-1}(R+Ci)} F(z) \exp(-\lambda_n \cdot \psi(z))\, \psi'(z)\, dz,$$

where

$$\psi(z) = \log |\exp (\psi(z))| + ci$$

and integration being taken along the path $\log |\exp (\psi(z))| = R$, *also, whene the limit* $C \to \pi$, *is considered,* $\lambda_n s$ *are taken to be integers. The expressions for* a_n *in case of Taylor series and Dirichlet series follow from the expression obtained here.*

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