Riv. Mat. Univ. Parma (2) **10** (1969), 101-109

A. B. MATHUR (*)

Some Results Involving Legendre's and F_4-Functions. (**)

1. - Introduction.

The LAPLACE transform, the HANKEL transform and the MEIJER transform of the function $f(t)$ are given by the following integral equations ($\operatorname{Re} p > 0$):

$$\varphi(p) = \int_0^\infty e^{-pt} f(t)\, dt \,, \tag{1.1}$$

$$\varphi(p) = \int_0^\infty (pt)^{1/2} J_\nu(pt) f(t)\, dt \,, \tag{1.2}$$

$$\varphi(p) = \int_0^\infty (pt)^{1/2} K_\nu(pt) f(t)\, dt \,. \tag{1.3}$$

We shall denote (1.1), (1.2) and (1.3) symbolically as $\varphi(p) \doteqdot f(t)$, $\varphi(p) \underset{\nu}{\overset{J}{=}} f(t)$ and $\varphi(p) \underset{\nu}{\overset{K}{=}} f(t)$ respectively.

The following property of HANKEL transform as given by SNEDDON will also be used:

If the functions $f(t)$ and $g(t)$ are such that the integrals $\int_0^\infty f(t)\, dt$ and $\int_0^\infty g(t)\, dt$ are absolutely convergent, and if the functions $f(t)$ and $g(t)$ are of bounded variations in the neighbourhood of the point t, and if $\varphi(p)$ and $\psi(p)$ denote their HANKEL transforms of order $\nu \geqslant -1/2$, then

---

(*) Indirizzo: Department of Mathematics, S. A. T. Institute, Vidisha (M. P.), India.

(**) Ricevuto: 9-IX-1968.

$$(1.4)\qquad \int_0^\infty f(t)\, g(t)\, dt = \int_0^\infty \varphi(t)\, \psi(t)\, dt .$$

2. – Theorem. *If*

$$(2.1)\qquad \varphi(p) \doteqdot f(t) ,$$

and

$$(2.2)\qquad \psi(p) \underset{\varrho}{\overset{K}{=}} t^{-1/2} K_\varrho(\alpha t)\, f(t),$$

then:

$$(2.3)\quad \left\{ \begin{array}{l} \psi(p) = \pi\, (4\, \alpha)^{(\varrho/2)-(1/4)} \cdot p^{(\varrho/2)+(1/4)} \cdot \\ \cdot \displaystyle\int_0^\infty \left\{(2\alpha + t)\, (2p + t)\right\}^{-(1/4)(2\varrho+1)} P_{\varrho-(1/2)} \left[\frac{(t+2\alpha)(t+2p)}{2\alpha p} - 1\right] \varphi(t+\alpha+p)\, dt , \end{array} \right.$$

provided $\operatorname{Re}\varrho > 0$, $\operatorname{Re}(p+\alpha) > 0$, $\operatorname{Re}(A+1) > 0$, $|\arg\alpha| < \pi$, $|\arg\beta| < \pi$ *and the integral involved in* (2.3) *is absolutely convergent. Here* $f(t) = O(|t|^A)$ *for small* t *and* $f(t) = O(|t|^B)$ *for large* t.

Proof. We have ([2], p. 285, (60))

$$(2.4)\quad \left\{ \begin{array}{l} e^{(\alpha+\beta)p} K_\varrho(\alpha p)\, K_\varrho(\beta p) \doteqdot \\ \doteqdot \pi\, (4\, \alpha\, \beta)^{(\varrho/2)-(1/4)} \left\{(2\alpha+t)(2\beta+t)\right\}^{-(1/4)(2\varrho+1)} P_{\varrho-(1/2)} \left[\dfrac{(t+2\alpha)(t+2\beta)}{2\, \alpha\, \beta} - 1\right] , \end{array} \right.$$

for $|\arg\alpha| < \pi$, $|\arg\beta| < \pi$ and $\operatorname{Re} p > 0$.

Applying PARSVAL's GOLDSTEIN theorem of LAPLACE transform to the relation (2.4) and the relation which we get after applying attentuation rule of operational calculus in (2.1), we get (2.3) with the help of the relation (2.2) after replacing β by p.

It may easily be seen that the integrals involved during the procedure are absolutely convergent under the conditions stated with the theorem.

Example 1. If we take $f(t) = t\, J_\mu(at)\, J_\mu(bt)$ then according to BAILEY [1]

$$\varphi(p) = \frac{(ab)^\mu\, \Gamma(2(\mu+1))}{2^{2\mu}\, \Gamma^2(\mu+1)\, p^{2(\mu+1)}}\; F_4\left[\mu+1, \frac{2\mu+3}{2};\ \mu+1,\ \mu+1;\ -\frac{a^2}{p^2}, -\frac{b^2}{p^2}\right] ,$$

provided $\operatorname{Re}(\mu+1)>0$, $\operatorname{Re} p>0$, $a>0$ and $b>0$. Also using a known result due to SAXENA [6], we get

$$\psi(p)=\frac{(ab)^{\mu}}{2\Gamma(\mu+1)}\sum_{\varrho,-\varrho}\frac{\alpha^{\varrho}\,p^{\varrho+(1/2)}\,\Gamma(-\varrho)\,\Gamma(\mu+\varrho+1)}{(\alpha^2+p^2+a^2+b^2)^{\mu+\varrho+1}}\cdot$$

$$\cdot F_4\left[\frac{\mu+\varrho+1}{2},\ \frac{\mu+\varrho+2}{2};\ \varrho+1,\ \mu+1;\ \frac{4\alpha^2p^2}{(\alpha^2+p^2+a^2+b^2)^2},\ \frac{4a^2b^2}{(\alpha^2+p^2+a^2+b^2)^2}\right],$$

provided $\operatorname{Re}(\mu\pm\varrho+1)>0$ and $\operatorname{Re}(\alpha+p)>|\operatorname{Im} a|+|\operatorname{Im} b|$.

Now substituting the values of $\varphi(p)$ and $\psi(p)$ in the relation (2.3), we get the following integral:

$$\int_0^\infty\frac{\{(2\alpha+t)(2p+t)\}^{-(1/4)(2\varrho+1)}}{(t+\alpha+p)^{2(\mu+1)}}\,P_{\varrho-(1/2)}\left[\frac{(2\alpha+t)(2p+t)}{2\alpha p}-1\right]\cdot$$

$$\cdot F_4\left[\mu+1,\ \frac{2\mu+3}{2};\ \mu+1,\ \mu+1;\ \frac{-a^2}{(t+\alpha+p)^2},\ \frac{-b^2}{(t+\alpha+p)^2}\right]dt=$$

$$=\frac{2^{2\mu-\varrho-(1/2)}\Gamma(\mu+1)}{\pi\,\Gamma(2(\mu+1))\,(\alpha p)^{(\varrho-1)/2}}\sum_{\varrho,-\varrho}\frac{(\alpha p)^{\varrho}\,\Gamma(-\varrho)\,\Gamma(\mu+\varrho+1)}{(\alpha^2+p^2+a^2+b^2)^{\mu+\varrho+1}}\cdot$$

$$\cdot\sum_{\varrho,-\varrho}F_4\left[\frac{\mu+\varrho+1}{2},\ \frac{\mu+\varrho+2}{2};\ \varrho+1,\ \mu+1;\ \frac{4\alpha^2p^2}{(\alpha^2+p^2+a^2+b^2)^2},\ \frac{4a^2b^2}{(\alpha^2+p^2+a^2+b^2)^2}\right],$$

provided $|\arg\alpha|<\pi$, $|\arg p|<\pi$, $\operatorname{Re}(\alpha+p)>0$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}(\mu+1)>0$, $a>0, b>0$, $\operatorname{Re}(\mu\pm\varrho+1)>0$, $\operatorname{Re}\varrho>0$ and $\operatorname{Re}(\alpha+p)>|\operatorname{Im} a|+|\operatorname{Im} b|$.

Example 2. If we take

$$f(t)=t\,I_\mu(at)\,I_\mu(bt),$$

then ([2], p. 196, (13))

$$\varphi(p)=\frac{(ab)^{\mu}\,\Gamma(2(\mu+1))}{2^{2\mu}\,p^{2(\mu+1)}\,\Gamma^2(\mu+1)}\,F_4\left[\mu+1,\ \frac{2\mu+3}{2};\ \mu+1,\ \mu+1;\ \frac{a^2}{p^2},\ \frac{b^2}{p^2}\right],$$

provided $\operatorname{Re}(2\mu+1)>0$ and $\operatorname{Re} p>|\operatorname{Re} a|+|\operatorname{Re} b|$. Also using a known result [4], we get

$$\psi(p) = (ab)^{\mu} \sum_{\varrho, -\varrho} \frac{p^{\varrho+(1/2)} \alpha^{\varrho} \Gamma(-\varrho) \Gamma(\mu+\varrho+1)}{(p^2+\alpha^2-a^2-b^2)^{\mu+\varrho+1}} \cdot$$

$$\cdot F_4 \left[\frac{\mu+\varrho+1}{2}, \frac{\mu+\varrho+2}{2}; \mu+1, \varrho+1; \frac{4\alpha^2 p^2}{(\alpha^2+p^2-a^2-b^2)^2}, \frac{4a^2b^2}{(\alpha^2+p^2-a^2-b^2)^2} \right],$$

provided $\mathrm{Re}(\mu \pm \varrho + 1) > 0$, $\mathrm{Re}(\mu+1) > 0$, $\mathrm{Re}\, a > 0$, $\mathrm{Re}\, b > 0$ and $\mathrm{Re}(\alpha + p) > \mathrm{Re}(a+b)$.

Now substituting the values of $\varphi(p)$ and $\psi(p)$ in the relation (2.3), we get the following integral:

$$\int_0^\infty \frac{\{(2\alpha+t)(2p+t)\}^{-(1/4)(2\varrho+1)}}{(t+\alpha+p)^{2(\mu+1)}} P_{\varrho-(1/2)} \left[\frac{(t+2\alpha)(t+2p)}{2\alpha p} - 1 \right] \cdot$$

$$\cdot F_4 \left[\mu+1, \frac{2\mu+3}{2}; \mu+1, \mu+1; \frac{a^2}{(t+\alpha+p)^2}, \frac{b^2}{(t+\alpha+p)^2} \right] dt =$$

$$= \frac{(ab)^{\mu} 2^{2\mu} \Gamma^2(\mu+1)}{\pi (4\alpha p)^{(\varrho/2)-(1/4)} \Gamma(2(\mu+1))} \sum_{\varrho, -\varrho} \frac{p^{\varrho+(1/2)} \alpha^{p} \Gamma(-\varrho) \Gamma(\mu+\varrho+1)}{(p^2+\alpha^2-a^2-b^2)^{\mu+\varrho+1}} \cdot$$

$$\cdot F_4 \left[\frac{\mu+\varrho+1}{2}, \frac{\mu+\varrho+2}{2}; \mu+1, \varrho+1; \frac{4\alpha^2 p^2}{(\alpha^2+p^2-a^2-b^2)^2}, \frac{4a^2b^2}{(\alpha^2+p^2-a^2-b^2)^2} \right],$$

provided $\mathrm{Re}(\mu \pm \varrho + 1) > 0$, $\mathrm{Re}(2\mu+1) > 0$, $\mathrm{Re}\, a > 0$, $\mathrm{Re}\, b > 0$, $\mathrm{Re}(\alpha+p) > > \mathrm{Re}(a+b)$.

Example 3. If we take

$$f(t) = t^{2m+2n-1} {}_0F_3\left(n+1, \frac{m+n}{2}, \frac{m+n+1}{2}; \frac{b^2t^4}{16} \right),$$

then according to ([2], p. 200, (19)), we get

$$\varphi(p) = \Gamma(2(m+n))\, p^{-2(m+n)} {}_2F_1\left(\frac{2m+2n+1}{4}, \frac{2m+2n+3}{4}, n+1; \frac{16b^2}{p^4} \right),$$

provided $\mathrm{Re}(m+n) > 0$ and $\mathrm{Re}\, p > \mathrm{Re}(2\sqrt{b})$. Also using a known result

([5], p. 94), we get

$$\psi(p) = \frac{\Gamma(m+n)}{2^{3-2m-2\varrho}} \sum_{\varrho,-\varrho} \frac{\Gamma(-\varrho)\,\Gamma(m+n+\varrho)\,\alpha^{\varrho}\,p^{\varrho+(1/2)}}{(p^2+\alpha^2)^{m+n+\varrho}} \cdot$$

$$\cdot\, F_4\left[\frac{m+n+\varrho}{2},\ \frac{m+n+\varrho+1}{2};\ \varrho+1,\ n+1;\ \frac{4\alpha^2p^2}{(\alpha^2+p^2)^2},\ \frac{16b^2}{(p^2+\alpha^2)^2}\right],$$

provided $\operatorname{Re}(m+n\pm\varrho)>0$ and $\operatorname{Re}(\alpha+p)>\operatorname{Re}(2\sqrt{b})$.

Now substituting the values of $\varphi(p)$ and $\psi(p)$ in the relation (2.3), we obtain the following integral:

$$\int_0^{\infty}\{(2\alpha+t)(2p+t)\}^{-(1/4)(2\varrho+1)}(p+\alpha+t)^{-2(m+n)}P_{\varrho-(1/2)}\left[\frac{(t+2\alpha)(t+2p)}{2\alpha p}-1\right]\cdot$$

$$\cdot\, {}_2F_1\left[\frac{2m+2n+1}{4},\ \frac{2m+2n+3}{4};\ n+1;\ \frac{16b^2}{(t+\alpha+p)^4}\right]dt =$$

$$= \frac{\Gamma(m+n)\,2^{2m+\varrho+(1/2)}}{\pi\,\Gamma(2(m+n))\,(\alpha p)^{(1/4)(2\varrho-1)}} \sum_{\varrho,-\varrho} \frac{\Gamma(-\varrho)\,\Gamma(m+n+\varrho)\,\alpha^{\varrho}\,p^{\varrho+(1/2)}}{(\alpha^2+p^2)^{m+n+\varrho}} \cdot$$

$$\cdot\, F_4\left[\frac{m+n+\varrho}{2},\ \frac{m+n+\varrho+1}{2};\ \varrho+1,\ n+1;\ \frac{4\alpha^2p^2}{(\alpha^2+p^2)^2},\ \frac{16b^2}{(\alpha^2+p^2)^2}\right],$$

provided $\operatorname{Re}(m+n\pm\varrho)>0$, $\operatorname{Re}(m+n)>0$, $\operatorname{Re}\alpha>0$ and $\operatorname{Re}(\alpha+p)>$ $>\operatorname{Re}(2\sqrt{b})$.

3. – In this section we have evaluated two integrals involving product of LEGENDRE's functions with the help of a well known property of HANKEL transform as given by SNEDDON ([7], p. 60).

Integral I:

$$(3.1)\quad \begin{cases} \displaystyle\int_0^\infty t\left\{\left(\frac{\alpha^2+b^2+t^2}{2bt}\right)^2-1\right\}^{-(\varrho/2)-(1/4)}\left\{\left(\frac{a^2+\beta^2+t^2}{2at}\right)^2-1\right\}^{(\varrho/2)-(1/4)}\cdot \\ \cdot\, Q^{\varrho+(1/2)}_{\nu-(1/2)}\left(\frac{\alpha^2+b^2+t^2}{2bt}\right)Q^{-\varrho+(1/2)}_{\nu-(1/2)}\left(\frac{a^2+\beta^2+t^2}{2at}\right)dt = \\ = \dfrac{\pi\, b^{\nu+\varrho+1}\beta^\varrho}{a^{\varrho-\nu-1}\alpha^\varrho\,\Gamma(\nu+1)}\sum_{\varrho,-\varrho}\dfrac{(\alpha\beta)^\varrho\,\Gamma(-\varrho)\Gamma(\varrho+\nu+1)}{(a^2+b^2+\alpha^2+\beta^2)^{\nu+\varrho+1}}\cdot \\ \cdot\, F_4\left[\dfrac{\varrho+\nu+1}{2},\dfrac{\varrho+\nu+2}{2};\varrho+1,\nu+1;\right. \\ \left.\dfrac{4\alpha^2\beta^2}{(a^2+b^2+\alpha^2+\beta^2)^2},\dfrac{4a^2b^2}{(a^2+b^2+\alpha^2+\beta^2)^2}\right], \end{cases}$$

provided $\mathrm{Re}(\nu\pm\varrho+1)>0$, $\mathrm{Re}(\nu+1)>0$, $\mathrm{Re}\,\alpha>|\mathrm{Im}\,b|$ and $\mathrm{Re}\,\beta>|\mathrm{Im}\,a|$.

Proof. Take ([4], p. 64, (12)):

$$(3.2)\quad \begin{cases} t^{\varrho+(1/2)}J_\nu(bt)\,K_\varrho(\alpha t)\underset{\nu}{\overset{J}{=}} \\ \underset{\nu}{\overset{J}{=}}\dfrac{\alpha^\varrho\, p^{-\varrho-(1/2)}e^{-\{\varrho+(1/2)\}\pi i}}{(2\pi)^{1/2}\,b^{\varrho+1}}\left\{\left(\dfrac{\alpha^2+b^2+p^2}{2bp}\right)^2-1\right\}^{-(\varrho/2)-(1/4)}Q^{\varrho+(1/2)}_{\nu-(1/2)}\left[\dfrac{\alpha^2+b^2+p^2}{2bp}\right], \end{cases}$$

provided $\mathrm{Re}\,\alpha>|\mathrm{Im}\,b|$, $\mathrm{Re}(\varrho+\nu+1)>0$ and $\mathrm{Re}(\nu+1)>0$.

Now replacing ϱ by $-\varrho$, b by a, and α by β, in the relation (3.2), we have:

$$(3.3)\quad \begin{cases} t^{-\varrho+(1/2)}J_\nu(at)\,K_\varrho(\beta t)\underset{\nu}{\overset{J}{=}} \\ \underset{\nu}{\overset{J}{=}}\dfrac{\beta^{-\varrho}\, p^{\varrho-(1/2)}e^{\{\varrho-(1/2)\}\pi i}}{(2\pi)^{1/2}\,a^{-\varrho+1}}\left\{\left(\dfrac{\beta^2+a^2+p^2}{2ap}\right)^2-1\right\}^{(\varrho/2)-(1/4)}Q^{-\varrho+(1/2)}_{\nu-(1/2)}\left[\dfrac{\beta^2+a^2+p^2}{2ap}\right], \end{cases}$$

provided $\mathrm{Re}\,\beta>|\mathrm{Im}\,a|$, $\mathrm{Re}(-\varrho+\nu+1)>0$ and $\mathrm{Re}(\nu+1)>0$.

Using the operational relations (3.2) and (3.3) in (1.4), we get (3.1) after evaluation the integral on the right with the help of the following known result [6]:

$$\int_0^\infty t\, J_\nu(at)\, J_\nu(bt)\, K_\varrho(\alpha t)\, K_\varrho(\beta t)\, dt =$$

$$= \frac{(ab)^\nu}{2\Gamma(\nu+1)} \sum_{\varrho,-\varrho} \frac{(\alpha\beta)^\varrho\, \Gamma(-\varrho)\, \Gamma(\varrho+\nu+1)}{(a^2+b^2+\alpha^2+\beta^2)^{\nu+\varrho+1}} \cdot$$

$$\cdot F_4\left[\frac{\varrho+\nu+1}{2},\ \frac{\varrho+\nu+2}{2};\ \varrho+1,\ \nu+1;\ \frac{4\alpha^2\beta^2}{(a^2+b^2+\alpha^2+\beta^2)^2},\ \frac{4a^2b^2}{(a^2+b^2+\alpha^2+\beta^2)^2}\right],$$

provided $\mathrm{Re}(\nu \pm \varrho + 1) > 0$ and $\mathrm{Re}(\alpha+\beta) > |\mathrm{Im}\, a| + |\mathrm{Im}\, b|$.

Integral II:

$$(3.4)\quad \begin{cases} \displaystyle\int_0^\infty t^{-1} \left\{\left(\frac{\alpha^2-b^2+t^2}{2bt}\right)^2+1\right\}^{-(\varrho/2)-(1/4)} \left\{\left(\frac{\beta^2-a^2+t^2}{2at}\right)^2+1\right\}^{(\varrho/2)-(1/4)} \cdot \\ \displaystyle\cdot\, Q^{\varrho+(1/2)}_{\nu-(1/2)}\left(\frac{b^2-\alpha^2-t^2}{2bti}\right) Q^{-\varrho+(1/2)}_{\nu-(1/2)}\left(\frac{a^2-\beta^2-t^2}{2ati}\right) dt = \\ \displaystyle = \frac{2\pi\, \beta^\varrho\, b^{\varrho+\nu+1}}{\alpha^\varrho\, a^{\varrho-\nu+1}}\, e^{\{(1/2)-\nu\}\pi i} \sum_{\varrho,-\varrho} \frac{(\alpha\beta)^\varrho\, \Gamma(-\varrho)\, \Gamma(\varrho+\nu+1)}{(\alpha^2+\beta^2-a^2-b^2)^{\varrho+\nu+1}} \cdot \\ \displaystyle\cdot\, F_4\left[\frac{\varrho+\nu+1}{2},\ \frac{\varrho+\nu+2}{2};\ \nu+1,\ \varrho+1;\ \frac{\alpha^2\beta^2}{(\alpha^2+\beta^2-a^2-b^2)^2},\ \frac{4a^2b^2}{(\alpha^2+\beta^2-a^2-b^2)^2}\right], \end{cases}$$

provided $\mathrm{Re}(\nu \pm \varrho + 1) > 0$, $\mathrm{Re}(\nu+1) > 0$, $\mathrm{Re}\,\alpha > 0$, $\mathrm{Re}\,\beta > 0$, $\mathrm{Re}\,\alpha >$ $> |\mathrm{Re}\, b|$ and $\mathrm{Re}\,\beta > |\mathrm{Re}\, a|$.

Proof. Take ([3], p. 66, (22)):

$$(3.5)\quad \begin{cases} \displaystyle t^{\varrho+(1/2)}\, I_\nu(bt)\, K_\varrho(\alpha t) \overset{J}{\underset{\nu}{=}} \\ \displaystyle \overset{J}{\underset{\nu}{=}} \frac{b^{-\varrho-1}}{(2\pi)^{1/2}}\, \alpha^\varrho\, p^{-\varrho-(1/2)}\, e^{-\{\varrho-(\nu/2)+(1/4)\}\pi i} \cdot \\ \displaystyle \cdot \left\{\left(\frac{\alpha^2-b^2+p^2}{2bp}\right)^2+1\right\}^{-(\varrho/2)-(1/4)} Q^{\varrho+(1/2)}_{\nu-(1/2)}\left(\frac{b^2-\alpha^2-p^2}{2bpi}\right), \end{cases}$$

provided $\mathrm{Re}\,(\nu+1) > 0$, $\mathrm{Re}\,(\nu+\varrho+1) > 0$ and $\mathrm{Re}\,\alpha > |\mathrm{Re}\, b|$.

Replacing ϱ by $-\varrho$, b by a and α by β in the above relation, we have

$$(3.6)\quad \begin{cases} t^{-\varrho+(1/2)} I_\nu(at) K_\varrho(\beta t) \underset{\nu}{\overset{J}{=}} \\ \underset{\nu}{\overset{J}{=}} \dfrac{a^{\varrho-1}}{(2\pi)^{1/2}} \beta^{-\varrho} p^{\varrho-(1/2)} e^{\{\varrho+(\nu/2)-(1/4)\}\pi i} \cdot \\ \cdot \left\{\left(\dfrac{\beta^2-a^2+p^2}{2ap}\right)^2+1\right\}^{(\varrho/2)-(1/4)} Q_{\nu-(1/2)}^{-\varrho+(1/2)}\left(\dfrac{a^2-\beta^2-p^2}{2api}\right), \end{cases}$$

provided $\mathrm{Re}(\nu+1)>0$, $\mathrm{Re}\,(\nu-\varrho+1)>0$ and $\mathrm{Re}\,\beta>|\,\mathrm{Re}\,a\,|$.

Using operational relations (3.5) and (3.6) in (1.4), we get (3.4) after evaluating the integral on the other side with the help of the following known result [4]:

$$\int_0^\infty t\, I_\nu(at)\, I_\nu(bt)\, K_\varrho(\alpha t)\, K_\varrho(\beta t)\, dt = (ab)^\nu \sum_{\varrho,-\varrho} \frac{(\alpha\beta)^\varrho\, \Gamma(-\varrho)\Gamma(\varrho+\nu+1)}{(\alpha^2+\beta^2-a^2-b^2)^{\varrho+\nu+1}} \cdot$$

$$\cdot F_4\left[\frac{\varrho+\nu+1}{2}, \frac{\varrho+\nu+2}{2}; \nu+1, \varrho+1, \frac{4\alpha^2\beta^2}{(\alpha^2+\beta^2-a^2-b^2)^2}, \frac{4a^2b^2}{(\alpha^2+\beta^2-a^2-b^2)^2}\right],$$

provided $\mathrm{Re}(\nu\pm\varrho+1)>0$, $\mathrm{Re}(\nu+1)>0$, $\mathrm{Re}\,\alpha>0$, $\mathrm{Re}\,\beta>0$ and $\mathrm{Re}(\alpha+\beta)>\mathrm{Re}(a+b)$.

The result (3.4) can be expressed as an infinite integral involving the product of two Gauss hypergeometric functions of real arguments with the help of the following relations:

$$Q_\nu^\mu(z) = \frac{e^{\mu\pi i}\pi^{1/2}\Gamma(\mu+\nu+1)}{2^{\nu+1}\Gamma(\nu+(3/2))} z^{-\mu-\nu-1}(z^2-1)^{(1/2)\mu}\, {}_2F_1\left(\frac{\mu+\nu+1}{2}, \frac{\mu+\nu+2}{2}; \nu+\frac{3}{2}; \frac{1}{z^2}\right).$$

I am grateful to Dr. P. M. Gupta, professor and head of the Department of Mathematics, for his valuable guidance and also to principal V. V. Natu for providing facilities.

References.

[1] W. N. BAILEY, *Some infinite integrals involving Bessel functions*, Proc. London Math. Soc. (2) **40** (1935), 37-48.

[2] A. ERDELYI, *Tables of Integral Transforms*, Vol. 1, McGraw-Hill, New York 1954.

[3] A. ERDELYI, *Tables of Integral Transforms*, Vol. 2, McGraw-Hill, New York 1954.

[4] A. B. MATHUR, *Some results involving Bessel's and F_4-functions* (communicated for publication).

[5] H. B. MALOO, Proc. Nat. Acad. Sci. India, Sec. A, **36** (1966), 89-96.

[6] R. K. SAXENA, *Integrals involving products of Bessel's functions*, Proc. Glassgow Math. Assoc. **6** (1964), 130-132.

[7] I. N. SNEDDON, *Fourier Transforms*, McGraw-Hill, New York 1951.

Abstract.

In this paper we have established one theorem on integral-transform and evaluated a few definite integrals involving products of Legendre's and F_4-functions. Later on we have also evaluated two interesting integrals involving product of two Legendre's functions.

* * *