

Rivista di Matematica della Università di Parma

Volume 1 - Number 1 - 2010

Proceeding of Seventh Meeting on

**Hyperbolic Conservation Laws and Fluid Dynamics:
Recent Results and Research Perspectives**
Trieste (Italy), August 31 - September 4, 2009

Edit by

Fabio Ancona
Stefano Bianchini
Rinaldo M. Colombo
Gianluca Crippa
Andrea Marson



Published by

UNIVERSITÀ DEGLI STUDI DI PARMA

Parma, Italy

Autorizzazione n. 47 del «Tribunale di Parma»
in data 31 dicembre 1949

Monograf S. R. L. Fotocomposizione
Via Collamarini 5, I-40138 Bologna
(tel.: 051 53 00 42)

Introduction

In the Italian research group on hyperbolic equations, fluid dynamics, dispersive equations and related topics, it has become a traditional appointment the annual conference held in SISSA, Trieste. This conference has been organized with several formats and goals in mind. In some cases it consists of 2/3 days of seminars, in order to have an up-to-date overview on the development of the research fields of our group. In other cases a bigger activity is planned, which can last one week and with courses of 4–6 hours and invited conferences.

The latter is the format of the conference held in SISSA, from August 31 to September 4, 2009: it consisted of 4 main courses and some invited contributions. Due to the success of the conference, the high number of participants and the quality of the contributions, the organizing committee decided to ask the main speakers to write an expanded note on the subject they presented during the activity, and these notes are collected in this volume.

The volume contains the expanded lecture notes of three courses:

- the course of L. C. Berselli on Navier-Stokes Equations with Navier Boundary Conditions,
- the course of C. Mascia on Shallow Water Equations,
- the course of L. V. Spinolo on ODE Techniques applied to Boundary Problems for Hyperbolic Systems of Conservation Laws,

and of the invited contributions:

- the seminar of F. Crispo on Zero Viscosity Limits of Navier-Stokes Equations,
- the seminar of F. Monti on L^∞ Solutions to 2×2 Hyperbolic Systems of Conservation Laws,
- the seminar of S. Spirito on the Navier-Stokes Equations with Artificial Viscosity.

The course of L. C. Berselli concerns the incompressible Navier-Stokes equations with slip boundary conditions. These boundary conditions are more natural than the usual no-slip condition under some physical situations, for example in the case of moderate pressure and low surface stresses, turbulent flows, or vanishing viscosity limits. In the last two cases the reason for which the no-slip boundary condition is believed to fail is that there is a thin boundary layer connecting the fluid variables in the interior of the domain with the boundary data, which becomes very thin in the limit of high turbulence or small viscosity, and only some part of the information at the boundary is transmitted to the fluid far from the boundary. These different boundary conditions pose different mathematical challenges, and require the development of new mathematical techniques. In these lectures the author presents an up-to-date overview of the problems and results of this new direction of research.

An introductory course on the subject of Shallow-Water equations and Saint-Venant model is given in the second contribution, the lecture notes of the course by C. Mascia. The aim is to present a basic introduction, but the author also provides the

details of the derivation of the model, well posedness, kinetic formulations, hydrodynamics limits, having in mind a non expert reader of this research field. Since C. Mascia is a very good lecturer and math communicator, the lectures have been very appreciated, and his notes contained in the volume are of particular clarity.

L. V. Spinolo is an expert on boundary conditions of conservation laws, but she chose to present the main techniques of singular ODEs and then applies them to particularly simple hyperbolic boundary problems. The main difficulty here is that the hyperbolic systems do not assume the boundary value: in fact one can think that a very thin boundary layer (where small scale terms play a role) connects the hyperbolic solution to the boundary data. Under natural assumptions, the key role is thus played by the ODE which describes travelling wave and boundary layers, and this study is usually performed by means of the invariant manifolds. In general these ODE are singular, and the author presents a self contained introduction to the analysis of these invariant manifolds for singular ODEs, and the application to boundary problems for hyperbolic systems of conservation laws.

The contribution of F. Crispo is a survey paper on the existence of solutions of Navier-Stokes equations with Navier boundary conditions, and its convergence to the zero viscosity limit. The results presented are the most recent ones, obtained by the research group on fluid-dynamics in Pisa, and it is an important complement of results and techniques given by the course of L. C. Berselli.

The case of large data for hyperbolic systems of conservation laws is one of the open problems in the 1 dimensional theory. The contribution of F. Monti describes a new proof of the Glimm-Lax result on the existence of a solution for the p -system of gas dynamics with L^∞ initial data. The main advantage of this proof is not only that cases other than the p -system are covered, but also that the proof has been considerably reduced in size and technicalities by means of recently developed techniques for hyperbolic systems.

The contribution of S. Spirito is related to the incompressible Navier-Stokes equations. A weak solution satisfying suitable admissibility conditions is constructed by allowing the fluid to be compressible, but with a compressibility constant that vanishes. The key part of the paper is to obtain bounds on the pressure and its gradient uniformly. This subject completes the survey on the new results and directions of the Italian fluid dynamics group located in Pisa.

The conference received a financial support by the Research Project MIUR PRIN 2007 “Sistemi non lineari di Leggi di Conservazione e Fluidodinamica”, by SISSA-ISAS and by the Department of Mathematics of University of Brescia.

Padova
Trieste
Brescia
Parma
Padova

Fabio Ancona
Stefano Bianchini
Rinaldo M. Colombo
Gianluca Crippa
Andrea Marson

Contents

LUIGI C. BERSELLI	
<i>Some results on the Navier-Stokes equations with Navier boundary conditions</i>	1–75
CORRADO MASCIA	
<i>A dive into shallow water</i>	77–149
LAURA V. SPINOLO	
<i>Notes on the study of the viscous approximation of hyperbolic problems via ODE analysis</i>	151–188
S. BIANCHINI, R. M. COLOMBO and F. MONTI	
<i>L^∞ solutions for 2×2 systems of conservation laws</i>	189–204
FRANCESCA CRISPO	
<i>On the zero-viscosity limit for 3D Navier-Stokes equations under slip boundary conditions</i>	205–217
STEFANO SPIRITO	
<i>Solutions of the Navier-Stokes equations constructed by artificial compressibility approximation are suitable</i>	219–230

