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On a non-existence result involving the fractional $p$-Laplacian

Abstract. We consider a nonlocal problem involving the fractional $p$-Laplacian operator in bounded smooth domains. A non-existence result is obtained via a comparison process. This result extends those done for the fractional Laplacian.

Keywords. Non-existence, weak solution, fractional $p$-Laplacian.

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1 - Introduction

Let $N \geq 1$ be an integer and let $\Omega$ be a bounded domain of $\mathbb{R}^N$. Consider the following nonlinear problem

\[
(E_p^s)
\left\{
\begin{array}{ll}
(\mathcal{A}_p^s) u &= f(x, u) & \text{in } \Omega, \\
u &= 0 & \text{in } \mathbb{R}^N \setminus \Omega.
\end{array}
\right.
\]

The nonlinear operator $(\mathcal{A}_p^s)$ with $p > 1$ and $s \in (0, 1)$ is the fractional $p$-Laplacian operator which is defined for any $x \in \mathbb{R}^N$ by

\[
(1) \quad (-\mathcal{A}_p^s) u(x) = 2 \lim_{\varepsilon \to 0} \int_{\mathbb{R}^N \setminus B_{\varepsilon}(x)} \frac{|u(x) - u(y)|^{p-2}}{|x-y|^{N+sp}} (u(x) - u(y)) dy.
\]

where $B_{\varepsilon}(x)$ denotes the open ball of radius $\varepsilon > 0$ and center $x$. System like $(E_p^s)$ can be met in the field of game theory; see [2] for a complete framework of the subject. When $s = 1$, the fractional $p$-Laplacian becomes the well-known $p$-Laplacian op-
erator. When $p = 2$, $(-\Delta)^s_p$ is reduced to be the linear operator $(-\Delta)^s$ usually called the fractional Laplacian and we obtain a semilinear problem which has been widely studied mainly to show non-existence results for nonlinear elliptic problems. In [15], X. Ros-Oton and J. Serra proved that $(E^s_p)$ admits no positive and bounded solution if $f$ is such that

$$\frac{N - 2s}{2N} uf(u) \geq \int_0^u f(t) dt, \quad \forall u \in \mathbb{R}. \quad (2)$$

This result is obtained by using a version of the Pohozaev identity for the fractional Laplace problem. Indeed, for $s = 1$, $p = 2$, one retrieves the well-known non-existence condition like (2) established by S. I. Pohozaev in his pioneer paper [14]. So, X. Ros-Oton and J. Serra generalized the results of [14] giving the fractional version of this identity. As an application of their work, they stated non-existence results for problem $(E^s_p)$ with supercritical nonlinearities $f$ in star-shaped domains $\Omega$. The results of [15] are developed in a full paper [18]. More recently, these authors extended their results showing in [17] the non-existence of nontrivial bounded solutions to some nonlinear problems in the form

$$\begin{cases}
Lu = f(u) \quad & \text{in } \Omega, \\
u = 0 \quad & \text{in } \mathbb{R}^N \setminus \Omega,
\end{cases} \quad (3)$$

where $L$ denotes a nonlocal operator in star-shaped domains $\Omega$. An operator is nonlocal in the sense that one needs the value of a function in all $\Omega$ and not only in a neighbourhood of a point, to determine the effect of the operator on it. X. Cabré and Y. Sire interested in [1], in the existence, variational properties and asymptotic behaviour of particular solutions to $(E^s_p)$ with $p = 2$. Nonlinear eigenvalues problems have been analyzed by A. Iannizzotto et al. in [10], showing the existence of eigenvalues. They established the existence of non-trivial weak solutions to $(E^s_p)$ with $f = \lambda |u|^{p-2}u$, so demonstrating the existence of fractional eigenvalues $\lambda$. Then using Morse theory, they proved the existence of non-zero solutions in the $p$-superlinear case $f$. M. D'Elia and M. Gunzburger studied in [3] discretization methods for the nonlocal operator $L$, the fractional Laplacian operator being a special case of $L$. The main contribution of their paper is the demonstration of the convergence of the nonlocal operator $L$ to the fractional Laplacian $-\Delta^s$ on bounded domains, under certain conditions. In [9], the eigenvalues of nonlocal operators, of which the fractional $p$-Laplacian operator is a particular case, were exploiting considering the weak solutions $u$ to the nonlocal problem. Paper [16] is devoted to the Pohozaev identity for the fractional Laplacian $(-\Delta)^s$ with $s > 1$. This paper extends the results of [18] and give as application, a continuation property for the fractional eigenfunctions. We can

When \( p > 1 \), the study of existence of solutions to fractional \( p \)-Laplacian problems with weight was the object of [12]. Indeed, this paper is concerned with the existence of solutions to

\[
\begin{align*}
(-\Delta)^s_p u &= \varphi(x)f(u) \quad \text{in } \mathbb{R}^N, \\
u &\geq 0, \quad u \neq 0.
\end{align*}
\]

In [6] A. Di Castro et al. investigated regularity results for nonlocal problem (3) with non-homogeneous Dirichlet condition, which can be reduced to problem \((E^s_p)\) in some cases. R. Ferreira and M. Pérez-Llanos went further with limit problems for fractional \( p \)-Laplacian. They described in [8] the behaviour of solutions to \((E^s_p)\), as \( p \to \infty \). A crucial paper for the theory of fractional Sobolev spaces \( W^{s,p} \), \( s \in (0,1) \), is [7], where the authors look the role of these spaces in the trace theory.

The present work aims to prove non-existence of nontrivial solutions for nonlinear problem \((E^s_p)\) in the case \( p > 1 \) and \( s \in (0,1) \).

Let \( \Omega \) be a regular bounded open set of \( \mathbb{R}^N \). Recall that

\[
C^{0,s}(\Omega) = \left\{ u \in C(\Omega) ; \sup_{x \neq y} \frac{|u(x) - u(y)|}{|x - y|^s} < \infty \right\},
\]

\[
C^{m,s}(\Omega) = \{ u \in C^m(\Omega), D^\beta u \in C^{0,s}(\Omega) , \forall \beta \text{ with } |\beta| = m \},
\]

where

\[
D^\beta u = \frac{\partial^{\beta} u}{\partial x_1^{\beta_1} \ldots \partial x_N^{\beta_N}} = \frac{\partial^{\beta_1 + \ldots + \beta_N} u}{\partial x_1^{\beta_1} \ldots \partial x_N^{\beta_N}}.
\]

Let \( \beta \) be a strictly positive real, \( k \) is the greater integer such that \( k < \beta \). \([\cdot ]_{C^{0,s-k}(\Omega)}\) designates the seminorm on the space \( C^{k,s-k}(\Omega) \),

\[
[u]_{C^{0,s-k}(\Omega)} = \sup_{x,y \in \Omega, x \neq y} \frac{|D^k u(x) - D^k u(y)|}{|x - y|^{s-k}}.
\]

The main result of this paper is as follows.

**Theorem 1.1.** Consider \( \Omega \) an open \( C^{1,1} \) bounded set of \( \mathbb{R}^N \). \( \delta(x) = \text{dist}(x, \partial \Omega) \). Let \( f \) be a locally Lipschitz function satisfying to

\[
\frac{N - sp}{Np} f(u) u \geq \int_0^u f(t) dt, \text{ for all } u \in \mathbb{R}.
\]

(4)
Assume that \( u \) is a \( W^{s,p}(\mathbb{R}^N) \) function which vanishes in \( \mathbb{R}^N \setminus \Omega \), and such that \( u \) is of class \( C^{0,1}(\Omega) \) and

\[
[u]_{C^{0,1}(\{x \in \Omega, \delta(x) \geq \rho\})} \leq C \rho^{s-1}, \quad \text{for all } \rho \in (0,1),
\]

then problem

\[
\begin{aligned}
\left\{ \begin{array}{l}
(-\Delta)^s_p u = f(u) & \quad \text{in } \Omega, \\
u = 0 & \quad \text{in } \mathbb{R}^N \setminus \Omega
\end{array} \right.
\end{aligned}
\]

admits no positive bounded solution. Moreover, if the inequality in (4) is strict, then problem (5) admits no nontrivial bounded solution.

**Corollary 1.1.** Assume that the hypotheses of Theorem 1.1 hold. If

\[
x \geq \frac{N(p-1)+sp}{N-sp},
\]

then problem

\[
\begin{aligned}
\left\{ \begin{array}{l}
(-\Delta)^s_p u = u|u|^{q-1} & \quad \text{in } \Omega, \\
u = 0 & \quad \text{in } \mathbb{R}^N \setminus \Omega
\end{array} \right.
\end{aligned}
\]

admits no positive bounded solution. Moreover, if \( x > \frac{N(p-1)+sp}{N-sp} \), then problem (6) admits no nontrivial bounded solution.

The paper is organized as follows. In Section 2, we give the weak formulation to problem \((E^s_p)\). Section 3 is devoted to the proof of the main result.

### 2 - Weak formulation

Let \( \Omega \) be an open set of \( \mathbb{R}^N \), star-shaped with respect to the origin of \( \mathbb{R}^N \) and let \( W^{s,p}(\mathbb{R}^N) \) be the fractional Sobolev space defined by

\[
W^{s,p}(\mathbb{R}^N) = \left\{ u \in L^p(\mathbb{R}^N) : u \text{ measurable, } \frac{|u(x) - u(y)|}{|x-y|^{N+sp}} \in L^p(\mathbb{R}^{2N}) \right\}
\]

endowed with the natural norm

\[
\|u\|_{W^{s,p}(\mathbb{R}^N)} = \left( \int_{\mathbb{R}^N} |u|^p dx + \int_{\mathbb{R}^{2N}} \frac{|u(x) - u(y)|^p}{|x-y|^{N+sp}} dxdy \right)^{\frac{1}{p}}.
\]
Consider the Gagliardo (semi-)norm of all measurable function $u : \mathbb{R}^N \to \mathbb{R}$:

$$[u]_{W^{s,p}({\mathbb{R}^N})} = \left( \int_{\mathbb{R}^{2N}} \frac{|u(x) - u(y)|^p}{|x - y|^{N+sp}} \, dx \, dy \right)^\frac{1}{p}.$$

See [7] for more details on the above notations. Now following [10], define the set

$$X(\Omega) = \{ u \in W^{s,p}(\mathbb{R}^N) : u(x) = 0 \text{ a.e. in } \mathbb{R}^N \setminus \Omega \},$$

which can be renormed by setting $\| \cdot \|_s = [\cdot]_{W^{s,p}(\mathbb{R}^N)}$. The dual space of $(X(\Omega), \| \cdot \|_s)$ is denoted by $(X(\Omega)^*, \| \cdot \|_s)$. Define for all $u, v \in X(\Omega)$, the nonlinear operator $A : X(\Omega) \to X(\Omega)^*$, by:

$$\langle A(u), v \rangle = \int_{\mathbb{R}^N} \int_{\mathbb{R}^N} \frac{|u(x) - u(y)|^{p-2}(u(x) - u(y))(v(x) - v(y))}{|x - y|^{N+sp}} \, dx \, dy$$

where here and in the rest of the paper, the notation $\langle \cdot, \cdot \rangle$ designates the duality brackets between the spaces $X(\Omega)^*$ and $X(\Omega)$.

If $u$ is smooth enough, this definition coincides with that of the fractional $p$-Laplacian (1). A (weak) solution of problem $(E_p^\lambda)$ is a function $u \in X(\Omega)$ such that for any $v \in X(\Omega)$,

$$\langle A(u), v \rangle = \int_{\Omega} f(x, u)v \, dx.$$

3 - Proof of the main result

Let $u$ be a function in $X(\Omega)$. Following [18], define in $\mathbb{R}^N$ the function:

$$u_\lambda(x) = u(\lambda x).$$

Since $u \equiv 0$ in $\mathbb{R}^N \setminus \Omega$, $\Omega$ is star-shaped then for $\lambda > 1$, it follows that $u_\lambda \equiv 0$ in $\mathbb{R}^N \setminus \Omega$. It follows that

$$\int_{\mathbb{R}^N} (- \Delta_p)^s u(x) u_\lambda(x) \, dx = \int_{\Omega} (- \Delta_p)^s u(x) u_\lambda(x) \, dx$$

and so,

$$\frac{d}{d\lambda} \bigg|_{\lambda=1} \int_{\mathbb{R}^N} u_\lambda(x)( - \Delta_p)^s u(x) \, dx = \frac{d}{d\lambda} \bigg|_{\lambda=1} \int_{\Omega} u_\lambda(x)( - \Delta_p)^s u(x) \, dx$$

where $\frac{d}{d\lambda} \bigg|_{\lambda=1}$ is the derivative at $\lambda = 1$. We set $g(x) = ( - \Delta_p)^s u(x)$ and taking account to the assumptions on $u$ and $f$, we refer us to the proof of Proposition 1.6, page
10 in [18] and so, we argue similarly. Hence, the following identity holds

$$ \int_{\Omega} (x \cdot \nabla u) A(u) dx = \frac{d}{d\lambda} \bigg|_{\lambda=1} \int_{\Omega} (-\Delta_p)^s u(x) u(z(x)) dx. $$

Then, we deduce

$$ \int_{\Omega} (x \cdot \nabla u) A(u) dx = \frac{d}{d\lambda} \bigg|_{\lambda=1} \int_{\mathbb{R}^N} (-\Delta_p)^s u(x) u(z(x)) dx. $$

Consequently, using (7) with $v = u_{\lambda}$, and making the changes of variables $z = \lambda^\frac{1}{p} x$ and $\omega = \lambda^\frac{1}{p} y$, we obtain

$$ \int_{\mathbb{R}^N} (-\Delta_p)^s u(x) u(z(x)) dx = \int_{\mathbb{R}^N} \int_{\mathbb{R}^N} \frac{|u(x) - u(y)|^{p-2}(u(x) - u(y))}{|x - y|^{N+sp}} (u(\lambda x) - u(\lambda y)) dxdy. $$

More precisely, we have

$$ \langle A(u), u \rangle = \int_{\mathbb{R}^{2N}} \frac{|u(\lambda^{-\frac{1}{p}}z) - u(\lambda^{-\frac{1}{p}}\omega)|^{p-2}}{|\lambda^{-\frac{1}{p}}z - \lambda^{-\frac{1}{p}}\omega|^{N+sp}} (u(\lambda^{-\frac{1}{p}}z) - u(\lambda^{-\frac{1}{p}}\omega)) $$

$$ \times (u(\lambda^{-\frac{1}{p}}z) - u(\lambda^{-\frac{1}{p}}\omega)) \lambda^{-\frac{2N}{p}} dzd\omega $$

$$ = \lambda^{\frac{2N}{p}} \int_{\mathbb{R}^{2N}} \frac{|u(\lambda^{-\frac{1}{p}}z) - u(\lambda^{-\frac{1}{p}}\omega)|^{p-2}}{|z - \omega|^{N+sp}} (u(\lambda^{-\frac{1}{p}}z) - u(\lambda^{-\frac{1}{p}}\omega)) $$

$$ \times (u(\lambda^{-\frac{1}{p}}z) - u(\lambda^{-\frac{1}{p}}\omega)) \lambda^{-\frac{2N}{p}} dzd\omega $$

$$ = \lambda^{\frac{2N}{p}} \int_{\mathbb{R}^{2N}} \left( \frac{|u(\lambda^{-\frac{1}{p}}z) - u(\lambda^{-\frac{1}{p}}\omega)|^{p-2}}{|\lambda^{-\frac{1}{p}}z - \lambda^{-\frac{1}{p}}\omega|^{N+sp}} \right)^{\frac{p}{p-2}} u(\lambda^{-\frac{1}{p}}z) - u(\lambda^{-\frac{1}{p}}\omega) $$

$$ \times \frac{u(\lambda^{-\frac{1}{p}}z) - u(\lambda^{-\frac{1}{p}}\omega)}{|\lambda^{-\frac{1}{p}}z - \lambda^{-\frac{1}{p}}\omega|^{N+sp}} dzd\omega $$

because we have remarked that $\lambda^{-\frac{p-2}{p}} N_{sp} + \frac{N_{sp}}{p} + \frac{N_{sp}}{p} = 1$ and $|z - \omega|^{N+sp} (p-2) + \frac{N_{sp}}{p} + \frac{N_{sp}}{p} = |z - \omega|^{N+sp}$.

Thus,

$$ (8) \quad \int_{\Omega} (x \cdot \nabla u) A(u) dx = \frac{sp - N}{p} \int_{\mathbb{R}^{2N}} \left( \frac{|u(z) - u(\omega)|}{|z - \omega|^{N+sp}} \right)^{p-2} \frac{u(z) - u(\omega)}{|z - \omega|^{N+sp}} $$

$$ \times \frac{u(z) - u(\omega)}{|z - w|^{N_{sp}}} dzd\omega + \frac{d}{d\lambda} \bigg|_{\lambda=1} I_{\lambda}, $$
where
\[
I_\lambda = \int_{\mathbb{R}^N} \left( \frac{u(\lambda^{-\frac{1}{r}}z) - u(\lambda^{-\frac{1}{r}}\omega)}{\lambda^{-\frac{1}{r}}z - \lambda^{-\frac{1}{r}}\omega} \right)^{p-2} \frac{u(\lambda^{-\frac{1}{r}}z) - u(\lambda^{-\frac{1}{r}}\omega)}{\lambda^{-\frac{1}{r}}z - \lambda^{-\frac{1}{r}}\omega} \frac{e^{-\frac{c_1}{\lambda^r}}}{\frac{N+sp}{r}} \times \frac{e^{-\frac{c_1}{\lambda^r}}}{\frac{N+sp}{r}} \lambda^{-\frac{1}{r}}z - \lambda^{-\frac{1}{r}}\omega \, dz\,dw.
\]

Then we can write
\[
(9) \quad \int_{\Omega} (x \cdot \nabla u) A(u) \, dx - \frac{sp - N}{p} \int_{\mathbb{R}^N} \frac{|u(x) - u(y)|^p}{|x - y|^{N+sp}} \, dxdy = \frac{d}{d\lambda} \bigg|_{\lambda = 1^+} I_\lambda.
\]

We have:
\[
\frac{d}{d\lambda} \bigg|_{\lambda = 1^+} I_\lambda = \lim_{\lambda \rightarrow 1^+} \frac{I_\lambda - I_1}{\lambda - 1},
\]

where
\[
I_1 = \int_{\mathbb{R}^N} \frac{|u(x) - u(y)|^p}{|x - y|^{N+sp}} \, dxdy.
\]

Let us study the sign of \(\frac{d}{d\lambda} \bigg|_{\lambda = 1^+} I_\lambda\). In view of the Cauchy-Schwarz inequality, we can write:
\[
I_\lambda \leq \int_{\mathbb{R}^N} \left( \frac{u(\lambda^{-\frac{1}{r}}z) - u(\lambda^{-\frac{1}{r}}\omega)}{\lambda^{-\frac{1}{r}}z - \lambda^{-\frac{1}{r}}\omega} \right)^{p-1} \frac{e^{-\frac{c_1}{\lambda^r}}}{\frac{N+sp}{r}} \frac{e^{-\frac{c_1}{\lambda^r}}}{\frac{N+sp}{r}} \lambda^{-\frac{1}{r}}z - \lambda^{-\frac{1}{r}}\omega \, dz\,dw.
\]

Set \(\phi_\lambda(z, \omega) = \frac{u(\lambda^{-\frac{1}{r}}z) - u(\lambda^{-\frac{1}{r}}\omega)}{\lambda^{-\frac{1}{r}}z - \lambda^{-\frac{1}{r}}\omega}\), then
\[
I_\lambda \leq \int_{\mathbb{R}^N} |\phi_\lambda(z, \omega)|^{p-1} |\phi_\lambda(\lambda z, \lambda \omega)| \, dz\,d\omega.
\]

But applying Hölder’s inequality for each \(\lambda > 1\) we get:
\[
(10) \quad I_\lambda \leq \left( \int_{\mathbb{R}^N} |\phi_\lambda(z, \omega)|^p \, dz\,d\omega \right)^{\frac{p-1}{p}} \left( \int_{\mathbb{R}^N} |\phi_\lambda(\lambda z, \lambda \omega)|^p \, dz\,d\omega \right)^{\frac{1}{p}}
\]

since \(\phi_\lambda(\cdot, \cdot) \in L^p\) (then \(\phi_\lambda(\cdot, \cdot)|^{p-1} \in L^{\frac{p}{p-1}}\)) and \(\phi_\lambda(\lambda \cdot, \lambda \cdot) \in L^p\). On the one hand, making the changes of variables \(x = \lambda^{-\frac{1}{r}}z\) and \(y = \lambda^{-\frac{1}{r}}\omega\), it follows that:
\[
\left( \int_{\mathbb{R}^N} |\phi_\lambda(z, \omega)|^p \, dz\,d\omega \right)^{\frac{p-1}{p}} \left( \int_{\mathbb{R}^N} \frac{|u(x) - u(y)|^p}{|x - y|^{N+sp}} \, dxdy \right)^{\frac{1}{p}}
\]

\[
= \lambda^{-\frac{N}{r}} \left( \int_{\mathbb{R}^N} \frac{|u(x) - u(y)|^p}{|x - y|^{N+sp}} \, dxdy \right)^{\frac{1}{p}}.
\]
On the other hand, making the changes of variables $X = \lambda^{\frac{1}{p}} z$ and $Y = \lambda^{\frac{1}{q}} \omega$, we get:

$$
\left( \int_{\mathbb{R}^{2N}} |\phi_\lambda(\lambda z, \lambda \omega)|^p dz d\omega \right)^{\frac{1}{p}} = \left( \lambda^{\frac{2N}{p}} \int_{\mathbb{R}^{2N}} \frac{|u(X) - u(Y)|^p}{|X - Y|^{N+sp}} dX dY \right)^{\frac{1}{p}}
$$

$$
= \lambda^{\frac{2N}{p} - \frac{1}{q}} \left( \int_{\mathbb{R}^{2N}} \frac{|u(X) - u(Y)|^p}{|X - Y|^{N+sp}} dX dY \right)^{\frac{1}{p}}.
$$

Coming back to (10), we deduce that:

$$
I_\lambda \leq \int_{\mathbb{R}^{2N}} \frac{|u(x) - u(y)|^p}{|x - y|^{N+sp}} dx dy = I_1.
$$

Consequently, the quantity $\frac{I_\lambda - I_1}{\lambda - 1}$ is negative since $\lambda > 1$ and then, $\frac{d}{d\lambda} \big|_{\lambda = 1} I_\lambda$ is negative too. Since $u$ is a solution of (5), the identity holds

$$
(11) \quad \int_\Omega (x \cdot \nabla u) A(u) dx = \int_\Omega (x \cdot \nabla u) f(u) dx = -N \int_\Omega F(u) dx
$$

where $F(t) = \int_0^t f(\zeta) d\zeta$. And finally, it follows from (9) that:

$$
-N \int_\Omega F(u) dx - \frac{sp - N}{p} \int_\Omega f(u) u dx \leq 0,
$$

from which we conclude that:

$$
(12) \quad \int_\Omega N \left( \frac{N - sp}{Np} f(u) u - \int_0^u f(\zeta) d\zeta \right) dx \leq 0.
$$

Now setting $f(u) = u |u|^p$ in (12) yields

$$
\int_\Omega \left( \frac{N - sp}{Np} - \frac{1}{s+1} \right) |u|^{s+1} dx \leq 0,
$$

which completes the proof of Theorem 1.1.
Now consider that \( f(x, u) \) instead of \( f(u) \). The following result holds

**Corollary 3.1.** Consider \( \Omega \) an open bounded set of \( \mathbb{R}^N \) of class \( C^{1,1} \). Let \( f \) a function of class \( C^{0,1}_{\text{loc}}(\Omega \times \mathbb{R}) \). Assume that \( u \) is a \( W^{s,p}(\mathbb{R}^N) \) function which vanishes in \( \mathbb{R}^N \setminus \Omega \) and \( u \) of class \( C^{0,1}(\Omega) \).

Then if the domain \( \Omega \) is star-shaped, and the condition holds

\[
N - \frac{sp}{p} f(x, t)u \geq NF(x, t) + x \cdot F_x(x, t), \text{ for all } x \in \Omega \text{ and } t \in \mathbb{R},
\]

then problem \((E_p^u)\) admits no positive bounded solution. Moreover, if the inequality (13) is strict, then problem \((E_p^u)\) admits no nontrivial bounded solution.

**Proof.** Considering \( f(x, u) \) instead of \( f(u) \), then (11) can be rewritten in the form:

\[
\int_{\Omega} (x \cdot \nabla u) A(u) dx = \int_{\Omega} (x \cdot \nabla u) f(x, u) dx = -N \int_{\Omega} F(x, u) dx - \int_{\Omega} x \cdot F_x(x, u) dx.
\]

Consequently (9) is equivalent to

\[
N \int_{\Omega} F(x, u) dx + \int_{\Omega} x \cdot F_x(x, u) dx + \frac{sp}{p} f(x, u) udx \geq 0,
\]

where \( F(x, u) = \int_{0}^{u} f(x, \xi) d\xi. \)

Our study shows that the well-known classical non-existence results (see for instance \([4, 5, 13]\)) can be interpreted as a limiting case of the diffusion fractional. Moreover, they extend for a non-linear fractional system those obtained by X. Ros-Oton and J. Serra in \([18]\) for a linear fractional operator. However, the tools used in \([18]\) seem fail if we consider the nonlinear case in perspective to the establishing of a typical Pohozaev identity taking account the nonlinearity of the fractional \( p \)-Laplacian operator. This open question is actually investigated by the authors and it will be presented in a next paper. An another open problem is the following: Is it possible to obtain without regularity assumption a non-existence result of any solution belonging in \( X(\Omega) \cap L^\infty(\Omega) \)?

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References


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