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Some generating relations associated with multiple hypergeometric series (**)

1 - Introduction, definitions and preliminaries

A multiple Gaussian hypergeometric series [**15**] is a hypergeometric series in two or more variables which reduces to the familiar Gaussian hypergeometric series

$$ {}_2F_1[a,b;\ c;\ z] = \sum_{n=0}^{\infty} \frac{(a)_n\ (b)_n}{(c)_n\ n!}\ z^n,\ (c \neq 0,-1,-2,\cdots), \tag{1.1}$$

whenever only one variable is non-zero. Here and throughout this work, the Pochhammer symbol $(a)_n$ is defined by

$$ (a)_n = \begin{cases} 1, & \text{if } n=0, \\ a(a+1)(a+2)\cdots(a+n-1), & \text{if } n=1,2,3\cdots. \end{cases} \tag{1.2}$$

The hypergeometric series in one or more variables occur frequently in a wide variety of problems in physics, mathematics, statistics, engineering sciences and operations research. The use of multiple hypergeometric series often facilitates the analysis by permitting complex expressions to be represented more simply in terms of some multi-variable function.

The general triple hypergeometric series (GTHS) $F^{(3)}[x,y,z]$ of Srivastava ([**13**];

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p. 428) is defined by

$$F^{(3)}[x,y,z] = F^{(3)}\left[\begin{array}{ll}(a) :: (b);(b');(b'') : (c);(c');(c'') & ; \\ & x,y,z \\ (e) :: (g);(g');(g'') : (h);(h');(h'') & ;\end{array}\right]$$

(1.3)

$$= \sum_{m,n,p=0}^{\infty} \frac{((a))_{m+n+p}((b))_{m+n}((b'))_{n+p}((b''))_{p+m}((c))_m((c'))_n((c''))_p \; x^m y^n z^p}{((e))_{m+n+p}((g))_{m+n}((g'))_{n+p}((g''))_{p+m}((h))_m((h'))_n((h''))_p \; m!\, n!\, p!},$$

where (a) and $((a))$ abbreviate the array of A parameters $a_1, a_2, , \cdots a_A$ and the product $\prod_{j=1}^{A}(a_j)$ respectively with similar interpretations for (b), $((b))$, (b'), $((b'))$, (b''), $((b''))$, *etc.*

The GTHS $F^{(3)}[x,y,z]$ is capable of unifying (and generalizing) the theory of triple Gaussian series ([**15**]; pp. 41-45). Further, we recall the definition of general double hypergeometric function or Kampé de Fériet function [1,7] in a slightly modified notation (see, for example, Srivastava and Panda ([17]; p. 423 (26)):

$$\text{(1.4)} \qquad F^{A:B;B'}_{C:D;D'}\left[\begin{array}{llll}(a) & :(b) & ;(b') & ; \\ & & & x,y \\ (c) & :(d) & ;(d') & ;\end{array}\right] = \sum_{m,n=0}^{\infty} \frac{((a))_{m+n}((b))_m((b'))_n \; x^m y^n}{((c))_{m+n}((d))_m((d'))_n \; m!\, n!}$$

for convergence of the series (1.1)-(1.4), see [**16**].

Also, we recall the following triple series ([**16**]; p. 62 (11))

$$\text{(1.5)} \qquad \Psi_2^{(3)}[a;\; b,c,d;\; x,y,z] = \sum_{m,n,p=0}^{\infty} \frac{(a)_{m+n+p}\; x^m\; y^n\; z^p}{(b)_m\; (c)_n (d)_p\; m!\, n!\, p!}$$

and note that $\Psi_2^{(2)} = \Psi_2$, where Ψ_2 denotes one of the Humbert's confluent hypergeometric functions of two variables.

A two-variable analogue of generalized Laguerre polynomials [**12**]

$$\text{(1.6)} \qquad L_m^{(a)}(x) = \sum_{r=0}^{m} \frac{(-1)^r \Gamma(m+a+1)\; x^r}{\Gamma(m-r+1)\Gamma(a+r+1)\; \Gamma(r+1)},$$

is given by Beniwal and Saran ([3]; p. 358 (1)) in the following form:

$$\text{(1.7)} \qquad L_{m,n}^{(a,\beta,\gamma)}(x,y) = \frac{(\beta)_m(\gamma)_n}{m!\; n!}\; F_2[a,-m,-n;\; \beta,\gamma;\; x,y],$$

where F_2 is Appell's function defined by ([16]; p. 53(5))

$$\text{(1.8)} \qquad F_2[a,b,b';c,c';\; x,y] = \sum_{m,n=0}^{\infty} \frac{(a)_{m+n}(b)_m(b')_n\; x^m y^n}{(c)_m(c')_n\; m!\; n!}, \;\; |\, x\,| + |\, y\,| < 1.$$

Also, we note that

$$\lim_{a\to\infty} L_{m,n}^{(a,\beta,\gamma)}\left(\frac{x}{a},\frac{y}{a}\right) = L_m^{(\beta-1)}(x)L_n^{(\gamma-1)}(y), \tag{1.9}$$

and

$$L_{m,n}^{(a,\beta,\gamma)}(0,y) = \frac{(\beta)_m}{m!}\, P_n^{(\gamma-1,a-\gamma-n)}(1-2y), \tag{1.10}$$

where $P_n^{(a,\beta)}(x)$ denotes Jacobi polynomials [**12**].

$$P_n^{(a,\beta)}(x) = \binom{a+n}{n}{}_2F_1\left[-n, a+\beta+n+1;\ a+1;\ \frac{1-x}{2}\right]. \tag{1.11}$$

Generating functions play an important role in the investigation of various useful properties of the sequences which they generate. Generating relations of special functions arise in a diverse range of applications in harmonic analysis, multivariate statistics, quantum physics, molecular chemistry and number theory.

Motivated by multifarious applications of multiple hypergeometric series and generating relations, in this paper, we derive generating relations involving GTHS $F^{(3)}[x,y,z]$ by using integral operators. A number of results involving double series and hypergeometric functions of Srivastava, Appell and Humbert are obtained as applications of these generating relations.

Now we recall some results involving generalized Laguerre polynomials.

The results given by Toscano [**18**]

$$\sum_{m=0}^{\infty} L_m^{(a+m)}(t)u^m = (1-4u)^{-1/2}\left(\frac{1+\sqrt{(1-4u)}}{2}\right)^{-a}\exp\left(-t\left(\frac{1-\sqrt{(1-4u)}}{1+\sqrt{(1-4u)}}\right)\right), \quad |\, u\,| < \frac{1}{4}, \tag{1.12}$$

and

$$\sum_{m=0}^{\infty} L_m^{(a-2m)}(t)u^m = (1+4u)^{-1/2}\left(\frac{1+\sqrt{(1+4u)}}{2}\right)^{a+1}\exp\left(\frac{-2tu}{1+\sqrt{(1+4u)}}\right), \quad |\, u\,| < \frac{1}{4}, \tag{1.13}$$

each of which is recorded also by (for example) Chen *et al.* ([**5**]; p. 358 (2.48, 2.49)) and Hansen ([**6**]; p. 318 (48.17.2), p. 319 (48.17.4)), where one can find many other related references including (for example) Rainville ([**12**]; p. 298 (152(3), p. 296 (150) (15)).

The results given by Brown [4]

$$\sum_{m=0}^{\infty} L_m^{(a-m/2)}(t)u^m = \frac{(1+w(u))^{1+a}}{\left(1+\frac{w(u)}{2}\right)} \exp(-tw(u)), \tag{1.14}$$

and

$$\sum_{m=0}^{\infty}\left(1+\frac{m}{2a}\right)^{-1} L_m^{(a-m/2)}(t)u^m = (1+w(u))^a \; {}_1F_1[2a;\; 2a+1;\; -tw(u)], \tag{1.15}$$

where $w(u) = \frac{u}{2}(u + \sqrt{(u^2+4)})$ and ${}_1F_1$ denotes confluent hypergeometric functions [12].

A result due to Srivastava [14] is given as:

$$L_m^{(a)}(\gamma t^2) = \sum_{r=0}^{m} \binom{a+m}{m-r}(\gamma)^r \; {}_2F_1[-m+r, a+r+1;\; a+r+1;\; \gamma]\, L_r^{(a)}(t^2). \tag{1.16}$$

Further, we recall the following results due to Bailey [2].

A result involving the product of generalized Laguerre polynomials with different exponents, but with the same order and argument, is given as:

$$L_m^{(a)}(z)L_m^{(\beta)}(z) = \frac{(1+a)_m(1+\beta)_m}{\Gamma(m+1)} \sum_{p=0}^{m} c_p L_{2p}^{(a+\beta)}(2z), \tag{1.17}$$

where

$$c_p = \frac{\left(\frac{1}{2}\right)_p}{(1+a)_p(1+\beta)_p\Gamma(m-p+1)} \; {}_3F_2\left[\begin{array}{lr} \frac{1}{2}(a+\beta+2p+1), \frac{1}{2}(a+\beta+2p+2), -m+p & ; \\ & 1 \\ a+p+1,\; \beta+p+1 & ; \end{array}\right],$$

and a trivial product of a classical generating function is given as:

$$\sum_{m,n=0}^{\infty} \frac{u^m\, v^n}{\Gamma(1+a+m)\Gamma(1+a+n)} L_m^{(a)}(z)L_n^{(a)}(z) = \exp(u+v)(uvz^2)^{-a/2} J_a(2(uz)^{1/2}) J_a(2(vz)^{1/2}), \tag{1.18}$$

where $J_\upsilon(x)$ denotes Bessel function [12]:

$$J_\upsilon(x) = \sum_{r=0}^{\infty} \frac{(-1)^r}{r!\,(\upsilon+r)!} \left(\frac{x}{2}\right)^{\upsilon+2r}. \tag{1.19}$$

The results (1.12)-(1.18) will be used to derive generating relations in the next section.

2 - Generating relations

We define the integral operator Ω by

$$\Omega_{\lambda,\upsilon}\{\ \} = \int_0^\infty t^\lambda \exp(-t^2) J_\upsilon(yt)\ \{\ \}\ dt.$$

In terms of this operator, we rewrite the following results which will serve as working tools in obtaining generating relations.

A result([**10**]; p. 186 (3)) involving Bessel function is given as:

$$\text{(2.1)}\quad \Omega_{\lambda,\upsilon}\{1\} = \frac{\Gamma\frac{1}{2}(\lambda+\upsilon+1)y^\upsilon}{\Gamma(\upsilon+1)2^{\upsilon+1}}\ {}_1F_1\left[\frac{1}{2}(\lambda+\upsilon+1);\upsilon+1;-y^2/4\right], \mathrm{Re}(\lambda+\upsilon) > -1,\ y\in R^+.$$

A result in [8] involving the product of two Laguerre polynomials and Bessel functions is given as:

$$\text{(2.2)}\quad \begin{aligned} \Omega_{\lambda,\upsilon}\{L_m^{(a)}(xt^2)\,L_n^{(\beta)}(zt^2)\} &= \frac{\Gamma\frac{1}{2}(\lambda+\upsilon+1)y^\upsilon}{\Gamma(\upsilon+1)2^{\upsilon+1}}\binom{m+a}{m}\binom{n+\beta}{n} \\ &\times F^{(3)}\left[\begin{matrix} \frac{1}{2}(\lambda+\upsilon+1) & :: \_;\_;\_ : \_\_; -m;\ -n & ; \\ & & -y^2/4, x, z \\ \_\_\_\_ & :: \_;\_;\_ : \upsilon+1; a+1; \beta+1 & ; \end{matrix}\right], \end{aligned}$$

an obvious special case of (2.2) is the following

$$\text{(2.3)}\quad \begin{aligned} \Omega_{\lambda,\upsilon}\{L_m^{(a)}(xt^2)\} &= \sum_{r=0}^\infty \frac{(-1)^r (y/2)^{\upsilon+2r}\Gamma(\frac{1}{2}(\lambda+\upsilon+1)+r)}{\Gamma(r+1)\Gamma(\upsilon+r+1)}\binom{m+a}{m} \\ &\times {}_2F_1\left[\frac{1}{2}(\lambda+\upsilon+1)+r, -m;\ a+1;\ x\right]. \end{aligned}$$

A result in [9] involving the product of three Bessel functions is given as:

$$\text{(2.4)}\quad \begin{aligned} \Omega_{\lambda,\upsilon}\{J_\sigma(xt)J_\delta(zt)\} &= \frac{x^\sigma\ y^\upsilon\ z^\delta\,\Gamma\frac{1}{2}(\lambda+\sigma+\upsilon+\delta+1)}{\Gamma(\sigma+1)\Gamma(\upsilon+1)\Gamma(\delta+1)(2)^{\sigma+\upsilon+\delta+1}} \\ &\times \Psi_2^{(3)}\left[\frac{1}{2}(\lambda+\sigma+\upsilon+\delta+1);\ \sigma+1,\upsilon+1,\delta+1;\ -x^2/4,-y^2/4,-z^2/4\right], \end{aligned}$$

$$\mathrm{Re}(\lambda+\sigma+\upsilon+\delta) > -1,\ x,y,z \in R^+.$$

In the process of deriving generating relations, we use the following operation:

($\boldsymbol{\rho}$) Multiplication by $t^{\lambda}\exp(-t^2)J_{\upsilon}(yt)$ and then integration with respect to t from $t=0$ to $t=\infty$.

We replace t by xt^2 in (1.12) and replace a, m and t by β, n and zt^2 respectively again in (1.12), and take the product of resultants, to obtain

$$\sum_{m,n=0}^{\infty} L_m^{(a+m)}(xt^2)L_n^{(\beta+n)}(zt^2)u^{m+n} = \theta^{-2}\left(\frac{1+\theta}{2}\right)^{-(a+\beta)}\exp\left(-(x+z)t^2\left(\frac{1-\theta}{1+\theta}\right)\right), \tag{2.5}$$

where $\theta=(1-4u)^{1/2}$ and $|\,u\,|<\dfrac{1}{4}$.

Now performing the operation (ρ) on (2.5) and using integrals (2.1) and (2.2), we obtain the following generating relation

$$\sum_{m,n=0}^{\infty}\binom{a+2m}{m}\binom{\beta+2n}{n}F^{(3)}\left[\begin{matrix}\psi & :: \_;\_;\_:\_; & -m\,; \quad -n & ; \\ & & & -Y^2, x, z \\ \_ & :: \_;\_;\_: & \phi; a+m+1; \beta+n+1 & ;\end{matrix}\right]u^{m+n}$$

$$= \theta^{-2}\left(\frac{1+\theta}{2}\right)^{-(a+\beta)}\left(1+(x+z)\left(\frac{1-\theta}{1+\theta}\right)\right)^{-\psi} {}_1F_1\left[\psi;\ \phi;\ \frac{-Y^2}{\left(1+(x+z)\left(\frac{1-\theta}{1+\theta}\right)\right)}\right], \tag{2.6}$$

where $\psi=(\lambda+\upsilon+1)/2$, $\phi=\upsilon+1$, $Y=y/2$ and $|\,u\,|<\dfrac{1}{4}$.

By using arguments similar to those used for obtaining generating relation (2.6), we obtain the following generating relations corresponding to (1.13), (1.14) and (1.15):

$$\sum_{m,n=0}^{\infty}\binom{a-m}{m}\binom{\beta-n}{n}F^{(3)}\left[\begin{matrix}\psi & :: \_;\_;\_:\_; & -m\,; \quad -n & ; \\ & & & -Y^2, x, z \\ \_ & :: \_;\_;\_: & \phi; a-2m+1; \beta-2n+1 & ;\end{matrix}\right]u^{m+n}$$

$$= \theta^{-2}\left(\frac{1+\eta}{2}\right)^{(a+\beta+2)}\left(1+(x+z)\left(\frac{2u}{1+\eta}\right)\right)^{-\psi} {}_1F_1\left[\psi;\ \phi;\ \frac{-Y^2}{(1+(x+z)\left(\frac{2u}{1+\eta}\right))}\right], \tag{2.7}$$

where $\eta=(1+4u)^{1/2}$ and $|\,u\,|<\dfrac{1}{4}$,

$$\sum_{m,n=0}^{\infty}\binom{a+m/2}{m}\binom{\beta+n/2}{n}F^{(3)}\left[\begin{matrix}\psi & :: \_;\_;\_:\_; & -m\,; \quad -n & ; \\ & & & -Y^2, x, z \\ \_ & :: \_;\_;\_: & \phi; a-\frac{m}{2}+1; \beta-\frac{n}{2}+1 & ;\end{matrix}\right]u^{m+n}$$

$$= (1+w(u))^{a+\beta+2}(1+w(u)/2)^{-2}(1+(x+z)w(u))^{-\psi} {}_1F_1\left[\psi;\ \phi;\ \frac{-Y^2}{(1+(x+z)w(u))}\right], \tag{2.8}$$

and

$$\sum_{m,n=0}^{\infty}\binom{a+m/2}{m}\binom{\beta+n/2}{n}\left(\frac{a\beta}{(a+m/2)(\beta+n/2)}\right)$$

$$\times F^{(3)}\left[\begin{matrix}\psi & :: \_;\_;\_ : \_; -m;\ -n & ; \\ & & -Y^2, x, z \\ \_ & :: \_;\_;\_ : \phi; a-m/2+1; \beta-n/2+1 & ;\end{matrix}\right]u^{m+n} \tag{2.9}$$

$$= (1+w(u))^{a+\beta}F^{(3)}\left[\begin{matrix}\psi & :: \_;\_;\_ : \_;\ 2a\ ;\ 2\beta & ; \\ & & -Y^2, xw(u), zw(u) \\ \_ & :: \_;\_;\_ : \phi; 2a+1; 2\beta+1 & ;\end{matrix}\right],$$

respectively.

Further replacing m, r, a and a by n, s, β and b respectively in (1.16) and taking the product of the resultant with (1.16) and then performing the operation (ρ) on this product and using integral (2.2), we obtain the following generating relation (after taking $\lambda = \upsilon + 1$)

$$\binom{m+a}{m}\binom{n+\beta}{n}F^{(3)}\left[\begin{matrix}\psi & :: \_;\_;\_ : \_; -m; -n & ; \\ & & -Y^2, \gamma, \gamma \\ \_ & :: \_;\_;\_ : \phi; a+1\,; \beta+1 & ;\end{matrix}\right]$$

$$= \sum_{r=0}^{m}\sum_{s=0}^{n}\binom{m+a}{m-r}\binom{n+\beta}{n-s}\binom{r+a}{r}\binom{s+b}{s}(\gamma)^{r+s}\,\frac{(b-\phi+1)_s}{(b+1)_s} \tag{2.10}$$

$$\times\, {}_2F_1[-m+r, a+r+1;\ a+r+1;\ \gamma]\, {}_2F_1[-n+s, b+s+1;\ \beta+s+1;\ \gamma]$$

$$\times F^{2:0;1}_{1:1;1}\left[\begin{matrix}\phi, \phi-b-1 & : \_ & ; -r & ; \\ & & & -Y^2, 1 \\ \phi-b-s-1 & : \phi & : a+1 & ;\end{matrix}\right].$$

Finally replacing z by zt^2 in (1.17) and (1.18) and then performing the operation (ρ) and by using integrals (2.2), (2.3) and (2.4), we obtain the following generating relations

$$F^{(3)}\left[\begin{matrix}\psi & :: \_;\_;\_ : \_;\ -m\ ;\ -m & ; \\ & & -Y^2, z, z \\ \_ & :: \_;\_;\_ : \phi; a+1\ ; \beta+1 & ;\end{matrix}\right]$$

$$= \sum_{r=0}^{\infty}\sum_{p=0}^{m} c_p \binom{a+\beta+2p}{2p}\,\frac{(\psi)_r(-Y^2)^r\Gamma(m+1)}{(\phi)_r\Gamma(r+1)}\,{}_2F_1[-2p, \psi+r;\ a+\beta+1;\ 2z] \tag{2.11}$$

and

$$\sum_{m,n=0}^{\infty} \frac{u^m\ v^n}{m!\ n!} F^{(3)} \left[\begin{matrix} \psi & :: \_;\_;\_ : \_; & -m\ ; & -n & ; \\ & & & & -Y^2, z, z \\ \_ & :: \_;\_;\_ : \phi; & a+1 & ; a+1 & ; \end{matrix} \right]$$

(2.12)

$$= \exp(u+v) \Psi_2^{(3)}[\psi;\ \phi, a+1, a+1;\ -Y^2, -uz, -vz].$$

We consider the applications of the results (2.6) and (2.12) in the next section.

3 - Applications of the main generating relations

I. Taking $\lambda = \upsilon + 1$ and $z = 1$ in (2.6), we obtain

$$\sum_{m,n=0}^{\infty} \binom{a+2m}{m} \frac{(\beta+n-\upsilon)_n}{n!} F^{2:0;1}_{1:1;1} \left[\begin{matrix} \upsilon+1, \upsilon-\beta-n & : \_\_ & ; & -m & ; \\ & & & & -Y^2, x \\ \upsilon-\beta-2n & : \upsilon+1 & ; & a+m+1 & ; \end{matrix} \right] u^{m+n}$$

(3.1)

$$= \theta^{-2} \left(\frac{1+\theta}{2} \right)^{-(a+\beta)} \left(1 + (x+1) \left(\frac{1-\theta}{1+\theta} \right) \right)^{-(\upsilon+1)} \exp \left(\frac{-Y^2}{\left(1 + (x+1) \left(\frac{1-\theta}{1+\theta} \right) \right)} \right), \mid u \mid < \frac{1}{4}.$$

Now, taking $y = 0$ in (3.1), we obtain

$$\sum_{m,n=0}^{\infty} \binom{a+2m}{m} \frac{(\beta+n-\upsilon)_n}{n!} {}_3F_2 \left[\begin{matrix} \upsilon+1, \upsilon-\beta-n, -m & ; \\ & x \\ \upsilon-\beta-2n, a+m+1 & ; \end{matrix} \right] u^{m+n}$$

(3.2)

$$= \theta^{-2} \left(\frac{1+\theta}{2} \right)^{-(a+\beta)} \left(1 + (x+1) \left(\frac{1-\theta}{1+\theta} \right) \right)^{-(\upsilon+1)}, \mid u \mid < \frac{1}{4}.$$

Further, taking $y = 0$ in (2.6), we obtain

$$\sum_{m,n=0}^{\infty} \binom{a+2m}{m} \binom{\beta+2n}{n} F_2[\psi, -m, -n;\ a+m+1, \beta+n+1;\ x, z] u^{m+n}$$

(3.3)

$$= \theta^{-2} \left(\frac{1+\theta}{2} \right)^{-(a+\beta)} \left(1 + (x+z) \left(\frac{1-\theta}{1+\theta} \right) \right)^{-\psi}, \mid u \mid < \frac{1}{4}.$$

By making use of (1.7), the generating relation (3.3) can be expressed as

$$\sum_{m,n=0}^{\infty} L_{m,n}^{(\psi, a+m+1, \beta+n+1)}(x, z) u^{m+n}$$

(3.4)

$$= \theta^{-2} \left(\frac{1+\theta}{2} \right)^{-(a+\beta)} \left(1 + (x+z) \left(\frac{1-\theta}{1+\theta} \right) \right)^{-\psi}, \mid u \mid < \frac{1}{4}.$$

Again, taking $x = 0$ in (3.4) and using (1.10) we obtain

$$\sum_{m,n=0}^{\infty} \binom{a+2m}{a+m} P_n^{(\beta+n,\psi-\beta-2n-1)}(1-2z)u^{m+n} = \theta^{-2}\left(\frac{1+\theta}{2}\right)^{-(a+\beta)}\left(1+z\left(\frac{1-\theta}{1+\theta}\right)\right)^{-\psi}, \tag{3.5}$$

where $| u |< \frac{1}{4}$.

II. Taking $y = 0$ in (2.12), we obtain

$$\sum_{m,n=0}^{\infty} \frac{u^m v^n}{m!\, n!} F_2[\psi, -m, -n; a+1, a+1; z, z] = \exp(u+v)\Psi_2^{(2)}[\psi; a+1, a+1; -uz, -vz]. \tag{3.6}$$

Taking $v = -u$ in (3.6) and using ([**15**]; p. 322 (188)), we obtain

$$\begin{aligned}&\sum_{m,n=0}^{\infty} \frac{(-1)^m u^{m+n}}{m!\, n!} F_2[\psi, -m, -n; a+1, a+1; z, z]\\ &= {}_2F_3\left[\begin{matrix} \frac{\psi}{2}, \frac{\psi+1}{2} & ; \\ & -u^2z^2 \\ a+1, \frac{a+1}{2}, \frac{a+2}{2} & ; \end{matrix}\right].\end{aligned} \tag{3.7}$$

Further setting $\psi = a+1$ in (3.6) and using ([**11**]; p. 452 (71)) and ([**15**]; p. 322 (182)), we obtain

$$\begin{aligned}&\sum_{m,n=0}^{\infty} \frac{u^m v^n}{m!\, n!} (1-z)^{m+n} {}_2F_1\left[-m, -n; a+1; \frac{z^2}{(1-z)^2}\right]\\ &= \exp((u+v)(1-z)) {}_0F_1[\_;\ a+1;\ uvz^2].\end{aligned} \tag{3.8}$$

Several other results can be obtained as applications of generating relations (2.7)-(2.11).

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Abstract

In this paper, a number of new generating relations are obtained for general triple hypergeometric series by using integral operators. Results involving double series and hypergeometric functions of Srivastava, Appell and Humbert are obtained as applications of these generating relations.

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