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Some multivariable bilateral generating relations involving Jacobi polynomials (**)

1 - Introduction

In the usual notation, let ${}_AF_B$ denotes generalized hypergeometric function of one variable with A numerator parameter and B denominator parameter, defined by [**18**], p. 73 (2)

$$ {}_AF_B\left[\begin{matrix}(a_A)\ ; \\ (b_B)\ ;\end{matrix}\ x\right] = 1 + \sum_{n=1}^{\infty} \frac{[(a_A)]_n x^n}{[(b_B)]_n n!} \tag{1.1}$$

where (a_A) denotes the array of A parameters given by $a_1, a_2, a_3, \ldots, a_A$ and Pochhammer symbol (or the shifted factorial, since $(1)_n = (n)!$) defined by

$$ [(b_B)]_n = \Pi_{m=1}^{B}(b_m)_n = \Pi_{m=1}^{B} \frac{\Gamma(b_m + n)}{\Gamma(b_m)} \tag{1.2}$$

the denominator parameters are neither zero nor negative integers, the numerator parameters may be zero or negative integers.

In 1921, Appell's four double hypergeometric functions ([**2**], p. 296 (1)) $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, $\mathbf{F}_4$ and its seven confluent forms $\boldsymbol{\Phi}_1$, $\boldsymbol{\Phi}_2$, $\boldsymbol{\Phi}_3$, $\boldsymbol{\Psi}_1$, $\boldsymbol{\Psi}_2$, $\boldsymbol{\Xi}_1$, $\boldsymbol{\Xi}_2$ were unified and generalized by Kampé de Fériet [**1**], p. 150 (29). We recall the defini-

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tion of general double hypergeometric function of Kampé de Fériet in the slightly modified notation of Srivastava and Panda [**31**], p. 423 (26):

$$\mathbf{F}^{\mathbf{A:B;D}}_{\mathbf{E:G;H}}\left[\begin{matrix}(a_A) : (b_B) \ ; (d_D) \ ; \\ (e_E) : (g_G) \ ; (h_H) \ ;\end{matrix} \ x, y\right] = \sum_{m,n=0}^{\infty} \frac{[(a_A)]_{m+n}[(b_B)]_m[(d_D)]_n x^m y^n}{[(e_E)]_{m+n}[(g_G)]_m[(h_H)]_n m!\, n!} \,. \tag{1.3}$$

Triple hypergeometric function $\mathbf{F}^{(3)}$ of Srivastava ([**21**], p. 428) is the unification and generalization of Lauricella's fourteen hypergeometric functions and is given by:

$$\begin{aligned} &\mathbf{F}^{(3)}\left[\begin{matrix}(a_A) :: (b_B) \ ; (d_D) \ ; (e_E) : (g_G) \ ; (h_H) \ ; (l_L) \ ; \\ (m_M) :: (n_N) \ ; (p_P) \ ; (q_Q) : (r_R) \ ; (s_S) \ ; (t_T) \ ;\end{matrix} \ x, y, z\right] \\ &= \sum_{i,j,k=0}^{\infty} \frac{[(a_A)]_{i+j+k}[(b_B)]_{i+j}[(d_D)]_{j+k}[(e_E)]_{k+i}[(g_G)]_i[(h_H)]_j[(l_L)]_k \ x^i \ y^j \ z^k}{[(m_M)]_{i+j+k}[(n_N)]_{i+j}[(p_P)]_{j+k}[(q_Q)]_{k+i}[(r_R)]_i[(s_S)]_j[(t_T)]_k \ i!\, j!\, k!} \,. \end{aligned} \tag{1.4}$$

In 1979, Exton defined the following double hypergeometric function

$$\begin{aligned} &\mathcal{H}^{A:B;C;D}_{E:G;M;N}\left[\begin{matrix}(a_A): \ (b_B); \ (c_C); \ \ (d_D); \\ (e_E): \ (g_G); \ (m_M); \ (n_N);\end{matrix} \ x, y\right] \\ &= \sum_{i,j=0}^{\infty} \frac{[(a_A)]_{2i+j}[(b_B)]_{i+j}[(c_C)]_i[(d_D)]_j}{[(e_E)]_{2i+j}[(g_G)]_{i+j}[(m_M)]_i[(n_N)]_j} \frac{x^i}{i!} \frac{y^j}{j!} \,. \end{aligned} \tag{1.5}$$

Making suitable adjustment in the numbers of numerator and denominator parameters of ([**18**], p. 73 (2)), we obtain Kampé de Fériet, double hypergeometric function given by

$$F^{B:C;D}_{G:M;N} = \mathcal{H}^{O:B;C;D}_{O:G;M;N}, \tag{1.6}$$

and another addional double hypergeometric function of Exton ([**9**], p. 137 (1.2)) given by

$$X^{A:C;D}_{E:M;N} = \mathcal{H}^{A:O;C;D}_{E:O,M;N}, \tag{1.7}$$

or, quivalently

$$\mathbf{X}^{\mathbf{A:B;D}}_{\mathbf{E:G;H}}\left[\begin{matrix}(a_A) : (b_B) \ ; (d_D) \ ; \\ (e_E) : (g_G) \ ; (h_H) \ ;\end{matrix} \ x, y\right] = \sum_{m,n=0}^{\infty} \frac{[(a_A)]_{2m+n}[(b_B)]_m[(d_D)]_n \ x^m \ y^n}{[(e_E)]_{2m+n}[(g_G)]_m[(h_H)]_n \ m!\, n!} \,, \tag{1.8}$$

which is the generalization and unification of Horn's non-confluent double hypergeometric function ([**10**]; see also 6, p. 225 (16)) $\mathbf{H}_4$ and Horn's confluent double hypergeometric function ([**11**]; see also 6, p. 226 (35)) $\mathbf{H}_7$.

Now we recall the definition of general quadruple hypergeometric function $\mathbf{F}^{(4)}$ of Srivastava ([**22**], p. 70 (2.5)) which is the generalization and unification of Srivastava's function ([**21**], p. 428) $\mathbf{F}^{(3)}$; Exton's quadruple hypergeometric functions [**7**] $\mathbf{K}_5, \mathbf{K}_9, \mathbf{K}_{10}, \mathbf{K}_{12}, \mathbf{K}_{13}, \mathbf{K}_{20}, \mathbf{K}_{21}$; Chandel's function ([**4**], p. 120 (2.3)) ${}^{(2)}_{(1)}E^{(4)}_{\mathbf{C}}$; Exton's functions ([8], p. 89 (3.4.1, 3.4.2)) ${}^{(2)}_{(1)}E^{(4)}_{\mathbf{D}}, {}^{(2)}_{(2)}E^{(4)}_{\mathbf{D}}$; Lauricella's functions ([**12**] $\mathbf{F}^{(4)}_{\mathbf{A}}, \mathbf{F}^{(4)}_{\mathbf{B}}, \mathbf{F}^{(4)}_{\mathbf{C}}, \mathbf{F}^{(4)}_{\mathbf{D}}$ and its confluent forms given by Exton; Erdélyi function ([**5**], p. 446 (7.2)) $\phi^{(4)}_2$; Humbert's function ([**11**], p. 429) $\mathbf{\Psi}^{(4)}_2$; Exton's functions ([8], p. 43 (2.1.1.4, 2.1.1.5)) $\mathbf{\Phi}^{(4)}_3, \mathbf{\Xi}^{(4)}_1$; Carlson's function of four variables ([**3**], p. 453 (2.1)) R; Srivastava-Exton function ([**28**], p. 373 (12, 13)) $\mathbf{\Phi}^{(4)}_{\mathbf{D}}$, et cetera. In our slightly modified notation, $\mathbf{F}^{(4)}$ is given by:

$$\tag{1.9} \begin{aligned} &\mathbf{F}^{(4)}\left[\begin{matrix}(a_A) :: (b_B);(d_D);(e_E);(g_G) : (h_H);(d_D);(m_M);(g_G) \,; \\ (n_N) :: (p_P);(q_Q);(r_R);(s_S) : (t_T);(q_Q)\,;(u_U)\,;(s_S)\,; \end{matrix}\; w,\, x,\, y,\, z\right] \\ &= \sum_{i,j,k,v=0}^{\infty} \frac{[(a_A)]_{i+j+k+v}[(d_D)]_{i+k}[(g_G)]_{j+v}[(b_B)]_i[(e_E)]_j[(h_H)]_k[(m_M)]_v\, w^i x^j y^k z^v}{[(n_N)]_{i+j+k+v}[(q_Q)]_{i+k}[(s_S)]_{j+v}[(p_P)]_i[(r_R)]_j[(t_T)]_k[(u_U)]_v\, i!\, j!\, k!\, v!}\,. \end{aligned}$$

Pathan's quadruple hypergeometric function ([**15**], p. 172 (1.2)) $\mathbf{F}^{(4)}_{\mathbf{P}}$ is the generalization and unification of Srivastava's function ([**21**], p. 428). In our modified notation, $\mathbf{F}^{(4)}_{\mathbf{P}}$ is given by:

$$\tag{1.10} \begin{aligned} &\mathbf{F}^{(4)}_{\mathbf{P}}\left[\begin{matrix}(a_A) :: (d_D);(g_G);(m_M);(q_Q) : (s_S);(v_V);(x_X);(z_Z)\,; \\ (b_B) :: (e_E);(h_H);\, (n_N);(r_R) : (t_T);(w_W);(y_Y);(u_U)\,; \end{matrix}\; C_1, C_2, C_3, C_4\right] \\ &= \sum_{i,j,k,l=0}^{\infty} \frac{[(a_A)]_{i+j+k+l}[(d_D)]_{i+j+k}[(g_G)]_{j+k+l}[(m_M)]_{k+l+i}[(q_Q)]_{l+i+j}[(s_S)]_i[(v_V)]_j}{[(b_B)_{i+j+k+l}[(e_E)]_{i+j+k}[(h_H)]_{j+k+l}[(n_N)]_{k+l+i}[(r_R)]_{l+i+j}[(t_T)]_i[(w_W)]_j} \\ &\quad\cdot \frac{[(x_X)]_k\, [(z_Z)]_l (C_1)^i\, (C_2)^j (C_3)^k (C_4)^l}{[(y_Y)]_k\, [(u_U)]_l\, i!\, j!\, k!\, l!}\,. \end{aligned}$$

Srivastava and Daoust defined extremely generalized hypergeometric function ([**25**]; p. 454 section 4, p. 456 (4.3); 26, p. 199 section 2; 27, pp. 157-158 section 5, pp. 153-157 sections (3, 4); 30, pp. 64-65 (18, 19, 20)) of n variables (which is referred to in the literature as the generalized Lauricella function of several variables). It is the generalization and unification of Srivastava's functions ([**21**], p. 428) $\mathbf{F}^{(3)}$, [**22**] $\mathbf{F}^{(4)}$; Pathan's functions [**15**] $F^{(4)}_P$, ([**16**], p. 51 (1)) and $\mathbf{F}^{(\mathbf{n}+1)}_{\mathbf{P}}$.Srivastava-Daoust defined the following series:

$$\mathbf{F}^{\mathbf{A}:\mathbf{B}^{(1)};\mathbf{B}^{(2)};\dots;\mathbf{B}^{(n)}}_{\mathbf{D}:\mathbf{E}^{(1)};\mathbf{E}^{(2)};\dots;\mathbf{E}^{(n)}}\left(\begin{matrix}[(a_A):\ \theta^{(1)},\ \theta^{(2)},\dots,\ \theta^{(n)}]:[(b^{(1)}_{B^{(1)}}):\ \Phi^{(1)}]\ ;\\ [(d_D):\ \Psi^{(1)},\ \Psi^{(2)},\dots,\ \Psi^{(n)}]:[(e^{(1)}_{E^{(1)}}):\ \delta^{(1)}]\ ;\end{matrix}\right.$$

$$\left.\begin{matrix}[(b^{(2)}_{B^{(2)}}):\ \Phi^{(2)}]\ ;\dots;\ [(b^{(n)}_{B^{(n)}}):\ \Phi^{(n)}]\ ;\\ [(e^{(2)}_{E^{(2)}}):\ \delta^{(2)}]\ ;\dots;\ [(e^{(n)}_{E^{(n)}}):\ \delta^{(n)}]\ ;\end{matrix}\ z_1,\ z_2,\dots,\ z_n\right) \tag{1.11}$$

$$= \sum_{m_1,\ m_2,\dots,\ m_n=0}^{\infty} \Omega(m_1,\ m_2,\dots,\ m_n)\ \frac{z_1^{m_1}}{(m_1)!}\ \frac{z_2^{m_2}}{(m_2)!}\cdots\frac{z_n^{m_n}}{(m_n)!}\ ,$$

where, for convenience

$$\Omega(m_1,\ m_2,\dots,\ m_n) = \frac{\prod_{j=1}^{A}(a_j)_{m_1\theta_j^{(1)}+m_2\theta_j^{(2)}+\dots+m_n\theta_j^{(n)}}\prod_{j=1}^{B^{(1)}}(b_j^{(1)})_{m_1\Phi_j^{(1)}}}{\prod_{j=1}^{D}(d_j)_{m_1\Psi_j^{(1)}+m_2\Psi_j^{(2)}+\dots+m_n\Psi_j^{(n)}}\prod_{j=1}^{E^{(1)}}(e_j^{(1)})_{m_1\delta_j^{(1)}}}$$

$$\cdot\frac{\prod_{j=1}^{B^{(2)}}(b_j^{(2)})_{m_2\Phi_j^{(2)}}\cdots\prod_{j=1}^{B^{(n)}}(b_j^{(n)})_{m_n\Phi_j^{(n)}}}{\prod_{j=1}^{E^{(2)}}(e_j^{(2)})_{m_2\delta_j^{(2)}}\cdots\prod_{j=1}^{E^{(n)}}(e_j^{(n)})_{m_n\delta_j^{(n)}}}\ , \tag{1.12}$$

the coefficients $\theta_j^{(k)}$, $j=1,2,\dots,A$; $\Phi_j^{(k)}$, $j=1,2,\dots,B^{(k)}$; $\Psi_j^{(k)}$, $j=1,2,\dots,D$; $\delta_j^{(k)}$, $j=1,2,\dots,E^{(k)}$; $\forall k\in\{1,2,\dots,n\}$ are zero and real constants (positive, negative)) ([**29**] pp. 270-272 (equations 5, 6, 7, 8, 9, 19, 20, 21)) and $(b^{(k)}_{B^{(k)}})$ abbreviates the array of $B^{(k)}$ parameters $b_j^{(k)}$, $j=1,2,\dots,B^{(k)}$; $\forall k\in\{1,2,\dots,n\}$, with similar interpretations.

2 - Some useful standard results

1 - *Binomial coefficients*

$$\binom{M}{N} = \frac{(M)!}{(N)!\,(M-N)!} = \binom{M}{M-N} \tag{2.1}$$

$$(-N)_K = \frac{(-1)^K N!}{(N-K)!}\ . \tag{2.2}$$

2 - *Jacobi and extended Jacobi (or Fujiwara) polynomials*

The classical Jacobi polynomial $P_N^{(A,\,B)}(x)$ of order (A,B) and degree N in x

defined (in terms of the Gauss hypergeometric ${}_2F_1$ function) by ([**18**], p. 255 (7))

$$(2.3)\quad P_N^{(A,\,B)}(x) = \frac{(1+A+B)_{2N}}{N!(1+A+B)_N}\left(\frac{x-1}{2}\right)^N {}_2F_1\left[\begin{matrix}-N,\ -A-N\ ; \\ \\ -A-B-2N\ ;\end{matrix}\ \frac{2}{1-x}\right]$$

which in view of ([**32**], p. 64), at once yields

$$(2.4)\qquad P_N^{(A,\,B)}(x) = \left(\frac{1-x}{2}\right)^N P_N^{(-A-B-2N-1,\,B)}\left(\frac{x+3}{x-1}\right)$$

$$(2.5)\qquad P_N^{(A,\,B)}(x) = \left(\frac{1+x}{2}\right)^N P_N^{(A,\,-A-B-2N-1)}\left(\frac{3-x}{1+x}\right).$$

An obvious variant of classical Jacobi polynomials is so called extended Jacobi polynomials $F_N^{(A,\,B)}(x;\,a,\,b,\,c)$ studied by (among others) Izuri Fujiwara in an attempt to give a unified presentation of classical orthogonal polynomials (especially Jacobi, Laguerre and Hermite polynomials) (see [**17**] and [**30**], p. 388, (1)). The polynomials $F_N^{(A,\,B)}(x;\,a,\,b,\,c)$ are essentially those that were considered by Szegö ([**32**], p. 58) and are given by ([**17**], p. 387)

$$(2.6)\qquad F_N^{(A,\,B)}(x;\,a,\,b,\,c) = [c(a-b)]^N P_N^{(A,\,B)}\left(\frac{2(x-a)}{a-b}+1\right).$$

3 - *Manocha's Result*

([**13**], p. 105 (1.4); see also [**30**], p. 122 (71))

$$(2.7)\quad \begin{aligned}\sum_{N=0}^{\infty}\binom{M+N}{N} P_{M+N}^{(\alpha-N,\beta-N)}(x)t^N &= \frac{(\alpha+\beta+M+1)_M}{M!}\left(\frac{x+1}{2}\right)^M\left(\frac{x+1}{x-1}\right)^\alpha \\ &\cdot\left\{1+\frac{1}{2}(x-1)\,t\right\}^{\alpha+\beta+M} {}_2F_1\left[\begin{matrix}-\alpha-\beta-M,\ -\alpha-M\ ; \\ \\ -\alpha-\beta-2M\ ;\end{matrix}\ \frac{2}{(x+1)\left\{1+\frac{1}{2}(x-1)\,t\right\}}\right].\end{aligned}$$

4 - *Lemma*

([**30**], p. 102 (17))

For positive integers $I_1, I_2, I_3, \ldots, I_j$, $(j \geq 1)$

$$\sum_{N=0}^{\infty} \sum_{K_1, K_2, \ldots, K_j = 0}^{I_1 K_1 + I_2 K_2 + \ldots + I_j K_j \leq N} C(K_1, K_2, \ldots K_j; N)$$

(2.8)

$$= \sum_{N, K_1, K_2, \ldots, K_j = 0}^{\infty} C(K_1, K_2, \ldots K_j; N + I_1 K_1 + I_2 K_2 + \ldots + I_j K_j).$$

3 - General multiple series identity

Theorem. *Let* $\{S(K_1, K_2, \ldots, K_j)\}$ *be a bounded multiple sequence of arbitrary complex numbers,* $\forall K_r \in \{0, 1, 2, 3, \ldots\}$, $r = 1, 2, \ldots, j$; $Z_1, Z_2, \ldots, Z_j$ *are complex variables,* $I_1, I_2, \ldots, I_j$, *and* M *are arbitrary non negative integers,* α *and* β *are arbitrary complex numbers then*

$$\sum_{N=0}^{\infty} \binom{M+N}{N} P_{M+N}^{(\alpha - N, \beta - N)}(x) \sum_{K_1, K_2, \ldots, K_j = 0}^{I \leq N} (-N)_I S(K_1, K_2, \ldots, K_j)$$

$$\frac{Z_1^{K_1}}{(K_1)!} \cdots \frac{Z_j^{K_j}}{(K_j)!} t^N = \frac{(\alpha + \beta + M + 1)_M}{(M)!} \left(\frac{x+1}{x-1} \right)^{\alpha} \left(\frac{x+1}{2} \right)^M$$

(3.1)
$$\left\{ 1 + \frac{1}{2}(x-1)\, t \right\}^{\alpha + \beta + M} \sum_{K_1, K_2, \ldots, K_j, K_{j+1} = 0}^{\infty} S(K_1, K_2, \ldots, K_j)$$

$$(-\alpha - \beta - M)_{I + K_{j+1}} \frac{(-\alpha - M)_{K_{j+1}}}{(-\alpha - \beta - 2M)_{K_{j+1}}} \left(\frac{xt - t}{2 + xt - t} \right)^I$$

$$\left[\frac{4}{(x+1)(2 + xt - t)} \right]^{k_{j+1}} \frac{Z_1^{K_1} Z_2^{K_2} \ldots Z_j^{K_j}}{(K_1)!(K_2)! \ldots (K_j)!(K_{j+1})!}$$

where for the sake of brevity $I = \sum_{m=1}^{j} I_m K_m$, $\forall K_r \in \{0, 1, 2, 3, \ldots\}$, $r = 1, 2, \ldots, j$; the variables $|Z_1|, |Z_2|, \ldots, |Z_j|$ also constrained that both sides of the expansion formula exist.

Proof of (3.1). In order to derive a class of extended bilateral generating relations for classical restricted Jacobi polynomials, we replace N by $N + I_1 K_1 + I_2 K_2 + \ldots + I_j K_j$ in left hand side «L» of Theorem (3.1) and use Lemma (2.8), to get

$$L = \sum_{N, K_1, K_2, \ldots, K_j = 0}^{\infty} \binom{M+N+I}{N+I} (-N-I)_I S(K_1, K_2, \ldots, K_j) \tag{3.2}$$
$$\frac{Z_1^{K_1} Z_2^{K_2} \ldots Z_j^{K_j}}{(K_1)!(K_2)! \ldots (K_j)!} t^{N+I} P_{M+I+N}^{(\alpha - I - N, \beta - I - N)}(x).$$

In view of the application of (2.1) and (2.2), we get

$$L = \frac{1}{M!} \sum_{K_1, K_2, \ldots, K_j = 0}^{\infty} (M+I)!(-t)^I S(K_1, K_2, \ldots, K_j) \frac{Z_1^{K_1} Z_2^{K_2} \ldots Z_j^{K_j}}{(K_1)!(K_2)! \ldots (K_j)!} \tag{3.3}$$
$$\sum_{N=0}^{\infty} \binom{M+I+N}{N} P_{M+I+N}^{(\alpha - I - N, \beta - I - N)}(x)\, t^N.$$

By the application of Manocha's result (2.7), we get

$$L = \frac{(\alpha + \beta + M + 1)_M}{(M)!} \left(\frac{x+1}{x-1} \right)^{\alpha} \left(\frac{x+1}{2} \right)^{M} \left\{ 1 + \frac{1}{2}(x-1)\, t \right\}^{\alpha + \beta + M}$$

$$\sum_{K_1, K_2, \ldots, K_j = 0}^{\infty} S(K_1, K_2, \ldots, K_j)(-\alpha - \beta - M)_I \frac{Z_1^{K_1} \ldots Z_j^{K_j}}{(K_1)! \ldots (K_j)!} \tag{3.4}$$

$${}_2F_1 \left[\begin{matrix} -\alpha - \beta - M + I,\ -\alpha - M\,; \\ \\ -\alpha - \beta - 2M\,; \end{matrix} \quad \frac{4}{(x+1)(2+xt-t)} \right] \left[\frac{xt - t}{2 + xt - t} \right]^I .$$

Now writing ${}_2F_1$ in power series, we get required right hand side of the theorem (3.1).

4 - Applications of (3.1)

The main result (3.1) of this paper offer many special cases of potential interests. Few are given in this section. In fact, by merely applying the relationship

(2.6), one can easily derive a number of results of Fujiwara's polynomials which appeared in the work of Pittaluga et al [**17**].

I. Setting $j = 2$, $I_1 = I_2 = 1$ and the bounded sequence

$$S(K_1, K_2) = \frac{[(a_A)]_{K_1+K_2}[(d_D)]_{K_1}[(g_G)]_{K_2}}{[(b_B)]_{K_1+K_2}[(e_E)]_{K_1}[(h_H)]_{K_2}}$$

in (3.1), we get a known generating relation of Srivastava ([**23**], p. 29 (3.4); see also [**19**], p. 40).

$$\sum_{N=0}^{\infty}\binom{M+N}{N} P_{M+N}^{(\alpha-N,\beta-N)}(x)\, \mathbf{F}_{\mathbf{B:E;H}}^{\mathbf{A+1:D;G}}\left[\begin{matrix}-N,\ (a_A):\ (d_D);\ (g_G); \\ \\ (b_B):\ (e_E);\ (h_H);\end{matrix}\ z_1, z_2\right] t^N$$

$$= \frac{(\alpha+\beta+M+1)_M}{M!}\left(\frac{x+1}{x-1}\right)^{\alpha}\left(\frac{x+1}{2}\right)^{M}\left\{1+\frac{1}{2}(x-1)\,t\right\}^{\alpha+\beta+M}$$

$$\mathbf{F}^{(3)}\left[\begin{matrix}(-\alpha-\beta-M) :: (a_A)\ ;\ : (d_D)\ ; (g_G)\ ; \\ \\ :: (b_B)\ ;\ : (e_E)\ ; (h_H)\ ;\end{matrix}\right.$$

$$\left.\begin{matrix}(-\alpha-M)\ ; \\ \\ (-\alpha-\beta-2M)\ ;\end{matrix}\ \frac{(xt-t)\,z_1}{(2+xt-t)},\ \frac{(xt-t)\,z_2}{(2+xt-t)},\ \frac{4}{(x+1)(2+xt-t)}\right]$$

which is the generalization and unification of bilateral generating relations of Manocha ([**13**], p. 105 (2.1); [**13**], p. 106 (2.2, 2.3)).

II. Setting $j = 2$, $I_1 = 1$, $I_2 = 0$ and using the bounded sequence given by (I), (3.1) reduces to a known bilateral generating relation of Srivastava ([**24**], p. 92 (6.5); see also [**20**], p. 692 (12); p. 694 (17, 18))

$$\sum_{N=0}^{\infty}\binom{M+N}{N} P_{M+N}^{(\alpha-N,\beta-N)}(x)\, \mathbf{F}_{\mathbf{B:E;H}}^{\mathbf{A:D+1;G}}\left[\begin{matrix}(a_A):\ -N,(d_D);\ (g_G); \\ (b_B):\ (e_E);\ (h_H);\end{matrix}\ z_1, z_2\right] t^N$$

$$= \frac{(\alpha+\beta+M+1)_M}{M!}\left(\frac{x+1}{x-1}\right)^{\alpha}\left(\frac{x+1}{2}\right)^{M}\left\{1+\frac{1}{2}(x-1)\,t\right\}^{\alpha+\beta+M}$$

$$\mathbf{F}^{(3)}\left[\begin{array}{l} :: (a_A) \;;(-\alpha-\beta-M \;:\; (d_D)\;;(g_G)\;; \\ \\ :: (b_B) \;; \qquad\qquad\qquad : (e_E)\;;(h_H)\;; \end{array}\right.$$

$$\left.\begin{array}{ll} (-\alpha-M) & ; \\ & \dfrac{(xt-t)z_1}{(2+xt-t)},\, z_2,\, \dfrac{4}{(x+1)(2+xt-t)} \\ (-\alpha-\beta-2M) & ; \end{array}\right].$$

III. For $j=2,\ I_1=1,\ I_2=0$ and the bounded sequence

$$S(K_1, K_2) = \frac{(\lambda)_{K_1+K_2}(\mu)_{K_2}}{(\delta)_{K_1+K_2}}\ ,\ \ z_1 = y,\, z_2 = z$$

(3.1) becomes

$$\sum_{N=0}^{\infty}\binom{M+N}{N} P_{M+N}^{(\alpha-N,\,\beta-N)}(x)\, F_1[\lambda,\ -N,\, \mu;\, \delta;\, y,\, z]\, t^N$$

$$= \frac{(\alpha+\beta+M+1)_M}{M!}\left(\frac{x+1}{x-1}\right)^{\alpha}\left(\frac{x+1}{2}\right)^{M}\left\{1+\frac{1}{2}(x-1)\,t\right\}^{\alpha+\beta+M}$$

$$F_M\Bigg[-\alpha-M,\, \lambda,\, \lambda,\ -\alpha-\beta-M,\, \mu,\ -\alpha-\beta-M;\ -\alpha-\beta-2M,\, \delta,\, \delta;$$

$$\frac{4}{(x+1)(2+xt-t)},\, z,\ \frac{(xt-t)\,y}{(2+xt-t)}\Bigg]$$

where F_M is a Lauricella's function of three variables defined by ([**30**]; p. 67)

$$F_M(\alpha_1,\, \alpha_2,\, \alpha_2,\, \beta_1,\, \beta_2,\, \beta_1;\, \gamma_1,\, \gamma_2,\, \gamma_2;\, x,\, y,\, z)$$

$$= \sum_{m,\,n,\,p=0}^{\infty} \frac{(\alpha_1)_m(\alpha_2)_{n+p}(\beta_1)_{m+p}(\beta_2)_n x^m y^n z^p}{(\gamma_1)_m(\gamma_2)_{n+p}\, m!\, n!\, p!}\,,$$

$$|x| < r\,, \quad |y| < s\,, \quad |z| < t\,, \quad r+t = 1 = s\,.$$

The result given above is a generalization of several other works on generating functions by Manocha and Sharma [**14**], Manocha [**13**] and Srivastava [**23**], [**24**].

IV. Setting $j=2,\ I_1=1,\ I_2=0$ and the bounded sequence

$$S(K_1, K_2) = \frac{(\lambda)_{K_1+K_2}(\mu)_{K_2}}{(\delta)_{K_2}(\gamma)_{K_1}}\,, \quad z_1 = y,\ z_2 = z$$

in equation (3.1), we get

$$\sum_{N=0}^{\infty} \binom{M+N}{N} P_{M+N}^{(\alpha-N,\,\beta-N)}(x)\, F_2[\lambda;\ -N, \mu;\ \gamma, \delta;\ y, z]\, t^N$$

$$= \frac{(\alpha+\beta+M+1)_M}{M!} \left(\frac{x+1}{x-1}\right)^{\alpha} \left(\frac{x+1}{2}\right)^{M} \left\{1 + \frac{1}{2}(x-1)\,t\right\}^{\alpha+\beta+M}$$

$$F_K\Big[\mu,\ -\alpha-\beta-M,\ -\alpha-\beta-M,\ \lambda,\ -\alpha-M,\ \lambda;\ \delta,\ -\alpha-\beta-2M,\ \gamma;\ z,$$

$$\frac{4}{(x+1)(2+xt-t)}\,,\ \frac{(xt-t)y}{(2+xt-t)}\Big]$$

where F_K is a Lauricella's function of three variables defined by [**30**]; p. 67

$$F_K(\alpha_1, \alpha_2, \alpha_2, \beta_1, \beta_2, \beta_1;\ \gamma_1, \gamma_2, \gamma_3;\ x, y, z)$$

$$= \sum_{m,\,n,\,p=0}^{\infty} \frac{(\alpha_1)_m(\alpha_2)_{n+p}(\beta_1)_{m+p}(\beta_2)_n x^m y^n z^p}{(\gamma_1)_m(\gamma_2)_n(\gamma_3)_p\, m!\, n!\, p!}\,,$$

$$|x| < r\,, \quad |y| < s\,, \quad |z| < t\,, \quad (1-r)(1-s) = t\,.$$

V. Setting $j=2,\ I_1=1,\ I_2=0$ and the bounded sequence

$$S(K_1, K_2) = \frac{(\lambda)_{K_1}(\gamma)_{K_2}(\delta)_{K_2}}{(\mu)_{K_1+K_2}}\,, \quad z_1 = y,\ z_2 = z$$

in equation (3.1), we get

$$\sum_{N=0}^{\infty} \binom{M+N}{N} P_{M+N}^{(\alpha-N,\,\beta-N)}(x)\, F_3[-N, \gamma, \lambda, \delta;\ \mu;\ y, z]\, t^N$$

$$= \frac{(\alpha+\beta+M+1)_M}{M!} \left(\frac{x+1}{x-1}\right)^{\alpha} \left(\frac{x+1}{2}\right)^{M} \left\{1 + \frac{1}{2}(x-1)\,t\right\}^{\alpha+\beta+M}$$

$$F_N\Big[-\alpha-M,\,\gamma,\,\lambda,\,-\alpha-\beta-M,\,\delta,\,-\alpha-\beta-M;\,-\alpha-\beta-2M,\,\mu,\,\mu;$$

$$\frac{4}{(x+1)(2+xt-t)},\,z,\,\frac{(xt-t)y}{(2+xt-t)}\Big]$$

where F_N is a Lauricella's function of three variables defined by [**30**]; p. 67

$$F_N(\alpha_1,\alpha_2,\alpha_3,\beta_1,\beta_2,\beta_1;\gamma_1,\gamma_2,\gamma_2;x,y,z)$$

$$=\sum_{m,n,p=0}^{\infty}\frac{(\alpha_1)_m(\alpha_2)_n(\alpha_3)_p(\beta_1)_{m+p}(\beta_2)_n x^m y^n z^p}{(\gamma_1)_m(\gamma_2)_{n+p}m!\,n!\,p!},$$

$$|x|<r,\quad |y|<s,\quad |z|<t,\quad (1-r)s+(1-s)\,t=0\,.$$

VI. Setting $j=2,\ I_1=2,\ I_2=1$ and the bounded sequence

$$S(K_1,K_2)=\frac{[(a_A)]_{2K_1+K_2}[(d_D)]_{K_1}[(g_G)]_{K_2}}{[(b_B)]_{2K_1+K_2}[(e_E)]_{K_1}[(h_H)]_{K_2}}$$

in equation (3.1), we get

$$\sum_{N=0}^{\infty}\binom{M+N}{N}P_{M+N}^{(\alpha-N,\,\beta-N)}(x)\,\mathbf{X}_{\mathbf{B:E;H}}^{\mathbf{A+1:D;G}}\left[\begin{matrix}-N,\ (a_A):\ (d_D);\ (g_G);\\ (b_B):\ (e_E);\ (h_H);\end{matrix}\ z_1,z_2\right]t^N$$

$$=\frac{(\alpha+\beta+M+1)_M}{M!}\left(\frac{x+1}{x-1}\right)^{\alpha}\left(\frac{x+1}{2}\right)^{M}\left\{1+\frac{1}{2}(x-1)\,t\right\}^{\alpha+\beta+M}$$

$$\mathbf{F}_{\mathbf{B:E;H;1}}^{\mathbf{A+1:D;G;1}}\left(\begin{matrix}[-\alpha-\beta-M\colon 2,1,1],\ [(a_A):2,1,0]:[(d_D):1];[(g_G):1];\\ [(b_B):2,1,0]:[(e_E):1];[(h_H):1];\end{matrix}\right.$$

$$\left.\begin{matrix}[-\alpha-M:1];\\ [-\alpha-\beta-2M:1];\end{matrix}\quad\frac{(xt-t)^2z_1}{(2+xt-t)^2},\,\frac{(xt-t)z_2}{(2+xt-t)},\,\frac{4}{(x+1)(2+xt-t)}\right).$$

VII. Taking $j=2,\ I_1=1,\ I_2=0$ and using the bounded sequence given by (VI), (3.1) takes the form

$$\sum_{N=0}^{\infty}\binom{M+N}{N} P_{M+N}^{(\alpha-N,\,\beta-N)}(x)\,\mathbf{X}_{\mathbf{B:E;H}}^{\mathbf{A:D+1;G}}\left(\begin{matrix}(a_A)\ :\ -N,(d_D)\ ;\ (g_G)\ ; \\ (b_B)\ :(e_E)\qquad ;\ (h_H)\ ;\end{matrix}\ z_1,\,z_2\right)t^N$$

$$=\frac{(\alpha+\beta+M+1)_M}{M!}\left(\frac{x+1}{x-1}\right)^{\alpha}\left(\frac{x+1}{2}\right)^{M}\left\{1+\frac{1}{2}(x-1)\,t\right\}^{\alpha+\beta+M}$$

$$\mathbf{F}_{\mathbf{B:E;H;1}}^{\mathbf{A+1:D;G;1}}\left(\begin{matrix}[-\alpha-\beta-2M:1,0,1],\ [(a_A):2,1,0]:[(d_D):1]; \\ [(b_B):2,1,0]:[(e_E):1];\end{matrix}\right.$$

$$\left.\begin{matrix}[(g_G):1];\qquad [-\alpha-M:1]; \\ [(h_H):1];\ [-\alpha-\beta-2M:1];\end{matrix}\ \frac{(xt-t)z_1}{(2+xt-t)},\ \frac{(xt-t)z_2}{(2+xt-t)},\ \frac{4}{(x+1)(2+xt-t)}\right).$$

VIII. If we set $j=2,\ I_1=2,\ I_2=1$ and

$$S(K_1,K_2)=\frac{[(a_A)]_{2K_1+K_2}}{[(b_B)]_{2K_1+K_2}}\,\frac{[(d_D)]_{K_1+K_2}[(g_G)]_{K_1}[(p_P)]_{K_2}}{[(e_E)]_{K_1+K_2}[(h_H)]_{K_1}[(q_Q)]_{K_2}}$$

in equation (3.1), we get

$$\sum_{N=0}^{\infty}\binom{M+N}{N} P_{M+N}^{(\alpha-N,\,\beta-N)}(x)\ \mathcal{H}_{B:E;H}^{A+1:D;G}\left[\begin{matrix}-N,\ (a_A):(d_D);\ (g_G);\ (p_P); \\ (b_B):(e_E);\ (h_H);\ (q_Q);\end{matrix}\ z_1,\,z_2\right]t^N$$

$$=\frac{(\alpha+\beta+M+1)_M}{M!}\left(\frac{x+1}{x-1}\right)^{\alpha}\left(\frac{x+1}{2}\right)^{M}\left\{1+\frac{1}{2}(x-1)\,t\right\}^{\alpha+\beta+M}$$

$$\mathbf{F}_{\mathbf{B+E:H;Q;1}}^{\mathbf{A+1+D:G;P;1}}\left(\begin{matrix}[-\alpha-\beta-M:2,1,1],[(a_A):2,1,0],[(d_D):1,1,0]: \\ [(b_B):2,1,0],[(e_E):1,1,0]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(g_G):1];[(p_P):1];\qquad [-\alpha-M:1]; \\ [(h_H):1];[(q_Q):1];\ [-\alpha-\beta-2M:1];\end{matrix}\ \frac{(xt-t)^2z_1}{(2+xt-t)^2},\ \frac{(xt-t)z_2}{(2+xt-t)},\ \frac{4}{(x+1)(2+xt-t)}\right).$$

IX. If $j=3$, $I_1 = I_2 = I_3 = 1$ and the bounded sequence

$$S(K_1, K_2, K_3) = \frac{[(a_A)]_{K_1+K_2+K_3}[(d_D)]_{K_1+K_2}[(g_G)]_{K_2+K_3}[(l_L)]_{K_3+K_1}[(q_Q)]_{K_1}}{[(b_B)]_{K_1+K_2+K_3}[(e_E)]_{K_1+K_2}[(h_H)]_{K_2+K_3}[(m_M)]_{K_3+K_1}[(r_R)]_{K_1}}$$

$$\frac{[(s_S)]_{K_2}[(v_V)]_{K_3}}{[(u_U)]_{K_2}[(w_W)]_{K_3}},$$

then equation (3.1) gives

$$\sum_{N=0}^{\infty}\binom{M+N}{N} P_{M+N}^{(\alpha-N,\,\beta-N)}(x)\,\mathbf{F}^{(3)}\left[\begin{matrix}-N,(a_A)::(d_D);\ (g_G);\ (l_L):(q_Q);\\ (b_B)::(e_E);\ (h_H);\ (m_M):(r_R);\end{matrix}\right.$$

$$\left.\begin{matrix}(s_S);(v_V);\\ (u_U);(w_W);\end{matrix}\ z_1, z_2, z_3\right] t^N$$

$$= \frac{(\alpha+\beta+M+1)_M}{M!}\left(\frac{x+1}{x-1}\right)^{\alpha}\left(\frac{x+1}{2}\right)^{M}\left\{1+\frac{1}{2}(x-1)t\right\}^{\alpha+\beta+M}$$

$$\mathbf{F}^{\mathbf{A+1+D+G+L;\,Q;\,S;\,V;\,1}}_{\mathbf{B+E+H+M:R;U;W;1}}\left(\begin{matrix}[-\alpha-\beta-M:1,1,1,1],[(a_A):1,1,1,0],[(d_D):1,1,0,0],\\ [(b_B):1,1,1,0],[(e_E):1,1,0,0],\end{matrix}\right.$$

$$\begin{matrix}[(g_G):0,1,1,0],\ [(l_L):1,0,1,0]:\ [(q_Q):1];\ [(s_S):1];\ [(v_V):1];\\ [(h_H):0,1,1,0],\ [(m_M):1,0,1,0]:\ [(r_R):1];\ [(u_U):1];\ [(w_W):1];\end{matrix}$$

$$\left.\begin{matrix}[-\alpha-M:1];\\ [-\alpha-\beta-2M:1];\end{matrix}\ \frac{(xt-t)z_1}{(2+xt-t)},\ \frac{(xt-t)z_2}{(2+xt-t)},\ \frac{(xt-t)z_3}{(2+xt-t)},\ \frac{4}{(x+1)(2+xt-t)}\right)$$

X. If $j=3$, $I_1 = I_2 = 1$, $I_3 = 0$ and the bounded sequence

$$S(K_1, K_2, K_3) = \frac{[(a_A)]_{K_1+K_2+K_3}[(d_D)]_{K_1+K_2}[(g_G)]_{K_2+K_3}[(l_L)]_{K_3+K_1}[(q_Q)]_{K_1}}{[(b_B)]_{K_1+K_2+K_3}[(e_E)]_{K_1+K_2}[(h_H)]_{K_2+K_3}[(m_M)]_{K_3+K_1}[(r_R)]_{K_1}}$$

$$\frac{[(s_S)]_{K_2}[(v_V)]_{K_3}}{[(u_U)]_{K_2}[(w_W)]_{K_3}}$$

in equation (3.1), we get

$$\sum_{N=0}^{\infty}\binom{M+N}{N} P_{M+N}^{(\alpha-N,\beta-N)}(x)\, \mathbf{F}^{(3)}\left[\begin{matrix}(a_A) :: -N,\ (d_D);\ (g_G);\ (l_L)\ :\ (q_Q); \\ (b_B) :: \qquad (e_E);\ (h_H);(m_M):(r_R);\end{matrix}\right.$$

$$\left.\begin{matrix}(s_S)\ ;(v_V)\ ; \\ (u_U);(w_W);\end{matrix}\ z_1,\, z_2,\, z_3\right] t^N$$

$$= \frac{(\alpha+\beta+M+1)_M}{M!}\left(\frac{x+1}{x-1}\right)^{\alpha}\left(\frac{x+1}{2}\right)^{M}\left\{1+\frac{1}{2}(x-1)t\right\}^{\alpha+\beta+M}$$

$$\mathbf{F}^{\mathbf{A+1+D+G+L:Q;S;V;1}}_{\mathbf{B+E+H+M:R;U;W;1}}\left(\begin{matrix}[-\alpha-\beta-M:1,1,0,1],[(a_A):1,1,1,0],[(d_D):1,1,0,0], \\ [(b_B):1,1,1,0],[(e_E):1,1,0,0],\end{matrix}\right.$$

$$\begin{matrix}[(g_G):0,1,1,0],\ [(l_L):1,0,1,0]:\ :[(q_Q):1];[(s_S):1];[(v_V):1]; \\ [(h_H):0,1,1,0],\ [(m_M):1,0,1,0]:[(r_R):1];[(u_U):1];[(w_W):1];\end{matrix}$$

$$\left.\begin{matrix}[-\alpha-M:1]; \\ [-\alpha-\beta-2M:1];\end{matrix}\ \frac{(xt-t)z_1}{(2+xt-t)},\ \frac{(xt-t)z_2}{(2+xt-t)},\ z_3,\ \frac{4}{(x+1)(2+xt-t)}\right).$$

For different values of $J, I_1, I_2, I_3, \ldots,$ and multiple bounded sequence $S(K_1, K_2, \ldots)$, we can derive a number of known and unknown bilateral generating relations involving Jacobi polynomials and Kampé de Feriet functions of two variables; Srivastava function $F^{(3)}$ and its special cases H_A, H_B, H_C, F_E, F_F, F_G, F_K, F_M, F_P, F_R, F_S, F_T; Srivastava function $F^{(4)}$ and its special cases $K_5, K_9, K_{10}, K_{12}, K_{13}, K_{20}, K_{21}$; Pathan function $F_P^{(4)}$ and its special cases K_2, K_{11}, K_{15}; Exton's function ${}^{(P)}H_3^{(n)}, {}^{(P)}H_4^{(n)}$, H, X; Wright's function ${}_p\psi_q$; Exton's function ${}^{(p)}_{(1)}E_D^{(n)}, {}^{(p)}_{(2)}E_D^{(n)}$; Chandel function ${}^{(p)}_{(1)}E_C^{(n)}$; Khichi function $H_B^{(n)}$; Karlsson's function $H_C^{(n)}$; Chandel-Gupta function ${}^{(p)}F_{AC}^{(n)}, {}^{(p)}F_{AD}^{(n)}, {}^{(p)}F_{BD}^{(n)}$; Karlsson's function ${}^{(p)}F_{CD}^{(n)}$; Exton's triple hypergeometric functions $X_1, X_2, X_3, \ldots, X_{20}$; Pandey function G_A, G_B; Dhawan function G_C, G_D; and Srivastava function G_C.

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Abstract

In this paper the authors prove a general theorem on generating relation for a certain sequence of functions. Many formulas involving the families of generating functions for the Jacobi and the so called extended Jacobi (or Fujiwara) polynomials given by Sharma and Manocha [**14**], *Manocha* [**13**], *Sharma* [**19**], *Sharma and Mittal* [**20**], *Manocha and Srivastava* [**23**], [**24**], *Pittaluga, Sacripante and Srivastava* [**17**] *are shown here to be special cases of a general class of a generating function involving Jacobi (or Fujiwara) polynomials and multiple hypergeometric series of several variables. It is then shown how the main result can be applied to derive a large number of generating functions involving hypergeometric functions of Appell, Lauricella, Kampé de Fériet, Srivastava, Pathan, Exton, Chandel, Khichi, Karlsson, Chandel-Gupta, Pandey, Dhawan and other multiple Gaussian hypergeometric functions scattered in the literature of special functions.*

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