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On *sg*-regular spaces (**)

1 - Introduction

In 1987, Bhattacharyya and Lahiri [**1**] introduced the semi-generalised closed (briefly *sg*-closed) set in a topological space and obtained its various properties. Since its introduction a large number of topologists like Dontchev, Maki [8], Caldas [4], Devi et al. [7], U. D. Tapi et al. [18] turned their attention to study different concepts of point set topology with the aid of *sg*-closed sets. Of these topologists, the contribution of Dontchev and Maki [8] deserves special mention. They have nicely solved two open problems left out by the second author and Lahiri [**1**]. The first one was: Whether the intersection of two *sg*-closed sets is *sg*-closed? The query has been elegantly met in [8] by proving that arbitrary intersection of *sg*-closed sets is *sg*-closed. This result, termed by us as Dontchev's Theorem, has been utilised in proving result (Theorem 5.5) in this paper.

The purpose of this paper is to introduce a new type of regular space termed *sg*-regular space with the help of *sg*-closed sets. In section **2** of this paper some known definitions and results necessary for the presentation of the subject are given. Section **3** deals with the definition and characterisation of *sg*-regular spaces. Section **4** is devoted to the investigation of subspaces and transformation of *sg*-regular spaces while the last section is concerned with some basic results of *sg*-regular spaces.

Throughout the paper (X, τ) and (Y, σ) (or simply X, Y) always denote the topological spaces. The closure (resp. interior) of the subset A is denoted by $Cl_X(A)$ (resp. $\mathrm{Int}_X(A)$) or simply by $Cl(A)$ (resp. $\mathrm{Int}(A)$)when there is no possibility of

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confusion. By A^C and (A, τ_A) we shall mean the complement of A and the subspace of the topological space (X, τ) respectively.

2 - Known definitions and results

Definition 2.1. In (X, τ), $A \subset X$ is called

(a) *a semi-open (briefly s.o.* [**11**]) *set iff there exists* $O \in \tau$ *such that* $O \subset A \subset Cl(O)$, *equivalently,* A *is s.o.* [**11**] *iff* $A \subset Cl\ (\mathrm{Int}(A))$;

(b) *a preopen (briefly p.o.* [**14**]) *set iff* $A \subset \mathrm{Int}(Cl(A))$;

(c) *a semi-generalised closed (briefly sg-closed* [**1**]) *set if* $scl(A) \subset O$ *whenever* $A \subset O$ *and* O *is semi-open in* X;

(d) *a semi-generalised open (briefly sg-open* [**1**]) *set if* A^c *is sg-closed.*

The family of all s.o. (resp. p.o., *sg*-closed) sets in X is denoted by $SO(X)$ (resp. $PO(X)$, $SGC(X)$). For each $x \in X$, the family of all s.o. (resp. p.o., *sg*-closed) sets containing x is denoted by $SO(X, x)$ (resp. $PO(X, x)$ $SGC(X, x)$). The family of all semi-closed (resp. preclosed, *sg*-open) sets in X is denoted by $SC(X)$ (resp. $PC(X)$, $SGO(X)$).

Definition 2.2. {[**2**], [**5**]}. For $A \subset X$, the semi-closure of A, denoted by $scl(A)$, is defined by $scl(A) = \cap \{B : B \in SC(X) \text{ and } B \supset A\}$.

Definition 2.3. ([**1**]). A space (X, τ) is semi-$T_{\frac{1}{2}}$ if every *sg*-closed set is semi-closed.

Definition 2.4. ([**12**]). A space (X, τ) is semi-R_0 iff for each semi-open set O and $x \in O$, $scl(\{x\}) \subset O$.

Definition 2.5. ([**10**]). A space (X, τ) is semi-regular if for every semi-closed set F and every point $x \notin F$, there exist $U \in SO(X, x)$, $V \in SO(X)$ such that $x \in U$, $F \subset V$ and $U \cap V = \phi$.

Definition 2.6. ([**18**]). A space (X, τ) is extremally disconnected if closure of every open set in X is open.

Definition 2.7. ([**19**]). (X, τ) is semi-compact if every semi-open cover of X admits a finite subcover.

Definition 2.8. ([**6**]). $f: X \to Y$ is semi-homeomorphism in the sense of Crossley and Hildebrand (briefly s.h.C.H.) if f is bijective, irresolute and presemi-open.

Definition 2.9. ([**6**]). A property preserved under semi-homeomorphism is called *a semi-topological property.*

Definition 2.10. ([**3**]). A subset N of a topological space X is said to be *a semi-neighbourhood of a point* $x \in X$ *if for some semi-open set* A *in* X, $x \in A \subset N$.

Lemma 2.1 (Lemma 2 [**16**]). *If* $x \in X$ *and* $A \subset X$, *then* $x \in scl(A)$ *iff*

$$A \cap U \neq \phi \qquad \forall U \in SO(X, x).$$

Theorem 2.1. ([**17**]). *If* $X_0 \subset X$ *and* $A \subset SO(X_0)$, *then* $A = B \cap X_0$ *for some* $B \in SO(X)$.

Theorem 2.2. ([**11**]). *If* $\{A_\alpha : \alpha \in \Lambda\}$ *be a family of s.o. sets in X then* $\bigcup_{\alpha \in \Lambda} A_\alpha \in SO(X)$.

Theorem 2.3. ([**15**]). *Let* $A \subset Y \subset X$ *such that* $Y \in SO(X)$. *Then* $A \in SO(X)$ *iff* $A \in SO(Y)$.

3 - Definition and characterisation of sg-regularity

Definition 3.1. A space X is termed sg-regular if for each $F \in SGC(X)$ and each point $x \notin F$, there exist U and $V \in SO(X)$ such that $x \in U$ and $F \subset V$ and $U \cap V = \phi$.

The families of all sg-regular spaces and semi-regular spaces are respectively denoted by $SGRS$ and SRS.

For the existence of sg-regular space, we see the following example.

Example 3.1. Let $X = \{a, b, c\}$ be endowed with the topology $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$. Then $SO(X) = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ and $SGC(X) = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$. It is easy to check that $(X, \tau) \in SGRS$.

Theorem 3.1. $SGRS \subset SRS$.

Proof. Since $SC(X) \subset SGC(X)$, the proof follows from Definition 3.1 and Definition 2.1.

Remark 3.1. The converse of the above theorem is not, in general, true as shown by

Example 3.2. Let $X = \{a, b, c\}$ be endowed with the topology $\tau = \{\phi, X, \{a\}, \{b, c\}\}$. Then (X, τ) is a regular space and hence a semi-regular space.

Also, $SGC(X) = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$. Now, $\{b\} \in SGC(X)$, $c \notin \{b\}$ but $\{c\}, \{b\}$ are not separated by s.o. sets. Hence, $(X, \tau) \in SRS$ but $(X, \tau) \notin SGRS$. The following theorems offers a characterisation of a *sg*-regular space.

Theorem 3.2. *Let X be a topological space, then the following statements are equivalent:*

(a) *X is sg-regular.*

(b) *For each x and each* $U \in SGO(X, x)$ *there exists* $V \in SO(X, x)$ *such that* $x \in V \subset scl(V) \subset U$.

(c) *For each* $A \in SGC(X)$, $A = \cap\{V : A \subset V$ *and V is semi-closed semi-neighbourhood of* $A\}$.

(d) *For each A and* $B \in SGO(X)$ *with* $A \cap B \neq \phi$, *there exists* $G \in SO(X)$ *such that*

(i) $A \cap G \neq \phi$, (ii) $scl(G) \subset B$.

(e) *For each* $A(\neq \phi)$ *and* $B \in SGC(X)$ *with* $A \cap B = \phi$ *there exist* U, $V \in SO(X)$ *with*

$$U \cap V = \phi \text{ such that (i) } A \cap V \neq \phi, \text{ (ii) } B \subset U.$$

Proof. (a) $\Rightarrow$ (b). Let $U \in SGO(X, x)$. Then $U^c \in SGC(X)$ and $x \notin U^c$. By *sg*-regularity of X there exist V, $W \in SO(X)$ such that $x \in V$, $U^c \subset W$ and $V \cap W = \phi$. Now the disjointness of V and W yields $V \subset W^c$ which, in its turn, implies that $scl(V) \subset W^c$. Thus, $V \subset scl(V) \subset W^c$. From this one infers that $x \in V \subset scl(V) \subset U$.

(b) $\Rightarrow$ (c). Assume $A \in SGC(X)$ and $x \notin A$. So, $A^c \in SGO(X)$. By (b), there exists $U \in SO(X, x)$ such that $x \in U \subset scl(V) \subset A^c$ whence $A \subset (scl(V))^c \subset U^c$. This shows that U^c is a semi-closed semi-neighbourhood of A which does not contain A. If we write $V = U^c$ then this implies that $A = \cap\{V : A \subset V$ and V is semi-closed semi-neighbourhood of $A\}$.

(c) $\Rightarrow$ (d). Let $A \cap B \neq \phi$ and $B \in SGO(X)$. Assume $x \in A \cap B$ then $B^c \in SGC(X)$ and $x \notin B^c$. Hence, by the argument applied in (c), there exists a semi-closed semi-neighbourhood V of B^c such that $x \notin V$ and $B^c \subset U \subset V$ where $U \in SO(X)$. Let $G = X - V$. Then $G \in SO(X, x)$ and $A \cap G \neq \phi$ which proves (i). Again $X - U \in SC(X)$. Consequently, $scl(G) \subset scl(X - V) \subset scl(X - U) = X - U \subset B$.

(d) $\Rightarrow$ (e). Assume $A \cap B = \phi \Rightarrow A \cap (X - B) = A \neq \phi$ and $X - B \in SGO(X)$. Then by (d), there exists a $G \in SO(X)$ such that $A \cap G \neq \phi$ and $G \subset scl(G) \subset X - B$. If we take $U = X - scl(G)$ then $B \subset U$. Also the construction of U gives $U \cap G = \phi$. Thus, there exist disjoint sets $U, G \in SO(X)$ with $A \cap G \neq \phi$ and $B \subset U$.

(e) $\Rightarrow$ (a). Suppose (e) holds. Let $x \notin B$ where $B \in SGC(X)$. Then $(\{x\}) \cap B = \phi$. Hence by (e), there exist $U, V \in SO(X)$ with $U \cap V = \phi$ such that $U \cap (\{x\}) \neq \phi$, $B \subset V$. This implies $x \in U$, $B \subset V$ which assures the sg-regularity of X.

4 - Subspaces and transformations

Let us first consider the following example.

Example 4.1. Let $X = \{a, b, c\}$ be endowed with the topology $\tau = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$. It is now easy to check that (X, τ) is sg-regular but (A, τ_A) is not sg-regular where $A = \{a, c\}$. Thus, any subspace of a sg-regular space is not sg-regular. However, the following holds.

Theorem 4.1. *If $A \in PO(X)$ and $A \in SGC(X)$ then (A, τ_A) is sg-regular if (X, τ) is so.*

To prove this theorem the following lemmas which are intersting in their own right are required. So, we state and prove them first.

Lemma 4.1. *If $A \in PO(X)$, $B \in SO(X)$ then $A \cap B \in SO(A)$.*

Proof. Let $B \in SO(X)$. Then there exists G open in X such that $G \subset B \subset Cl_X(G)$. From this it follows that $A \cap G \subset A \cap B \subset Cl_X(G) \cap A$. Preopenness of A then gives $G \cap A \subset A \cap B \subset Cl_X(G) \cap \mathrm{Int}_X(Cl_X(A)) \subset Cl_X(G \cap \mathrm{Int}_X(Cl_X(A))) \subset Cl_X(\mathrm{Int}_X(G \cap Cl_X(A))) \subset Cl_X(\mathrm{Int}_X(Cl_X(A \cap G))) \subset Cl_X(G \cap A)$. Now $G \cap A$ is open in A. We have, then $(G \cap A) \cap A \subset (A \cap B) \cap A \subset Cl_X(G \cap A) \cap A \Rightarrow G \cap A \subset (A \cap B) \subset Cl_A(G \cap A) \Rightarrow A \cap B \in SO(A)$.

Lemma 4.2. *If $B \subset Y \subset X$ and $Y \in PO(X)$ then $scl_X(B) \cap Y = scl_Y(B)$.*

Proof. Let $y \in scl_Y(B)$ so that $y \in Y$. Let $V \in SO(X, y)$. Then, by Lemma 4.1, $V \cap Y \in SO(Y, y)$. Consequently, by Lemma 2.1, $V \cap Y \cap B \neq \phi \Rightarrow V \cap B \neq \phi \Rightarrow y \in scl_X(B)$. Hence $scl_Y(B) \subset scl_X(B) \cap Y$. To prove the reverse inclusion, let $y \in scl_X(B) \cap Y \Rightarrow y \in scl_X(B)$, $y \in Y$. Let $V_0 \in SO(Y, y)$. Then, by Theorem 2.1, there exists $U \in SO(X)$ such that $V_0 = Y \cap U$. Also $U \in SO(X, y)$. So, $y \in scl_X(B) \Rightarrow U \cap B \neq \phi$. Therefore, $Y \cap U \cap B \neq \phi$. This indicates $V_0 \cap B \neq \phi \Rightarrow y \in scl_Y(B)$. Consequently, $scl_X(B) \cap Y \subset scl_Y(B)$. Hence, $scl_X(B) \cap Y = scl_Y(B)$.

Remark 4.1. The Lemma 4.2 is an improvment of the Lemma of Maheswari et al. [**13**] as openness of Y in the Lemma has been replaced by its preopenness.

Lemma 4.3. *If $B \subset A \subset X$, $A \in PO(X)$ and $A \in SGC(X)$ then $B \in SGC(A)$ iff $B \in SGC(X)$.*

Proof. **Necessity**. Suppose $B \in SGC(A)$. Then $scl_A(B) \subset O$ whenever $B \subset O \in SO(A)$. Since $O \in SO(A)$, by Theorem 2.1, $O = U \cap A$ for some $O \in SO(X)$. So, $scl_A(B) \subset U \cap A$ where $B \subset U \cap A \subset U$. This yields, by Lemma 4.2, $scl_X(B) \cap A \subset U \cap A$. Hence, $([scl_X(B)]^c) \cap A) \cup [scl_X(B) \cap A] \subset ([scl_X(B)]^c \cap A) \cup (U \cap A)$. Consequently, $A \cap X \subset ([scl_X(B)]^c) \cup U \Rightarrow A \subset V$ where $V = ([scl_X(B)]^c) \cup U \in SO(X)$. The *sg*-closedness of A in X then assures that $scl_X(A) \subset V$. Thus $scl_X(B) \subset scl_X(A) \subset V$ where $B \subset A \subset V$. This indicates that $B \in SGC(X)$.

Sufficiency. Assume $B \in SGC(X)$. Let $B \subset U$ where $U \in SO(A)$. Again, by Theorem 2.1, there exists $V \in SO(X)$ such that $U = A \cap V$. From this, it follows that $B \subset V$. The *sg*-closedness of B then produces that $scl_X(B) \subset V$ whence, by Lemma 4.2, $scl_A(B) = scl_X(B) \cap A \subset U$. This shows that $B \in SGC(A)$.

Remark 4.2. The Lemma 4.3 is an improvement of the Theorem 3 of Bhattacharyya et al. [**1**] as openness of A in that theorem has been replaced by its preopenness.

Proof of the Theorem 4.1. Suppose (X, τ) is a *sg*- regular space. Let $p \in A$ and $p \notin B \in SGC(A)$. Now the conditions (i) $B \subset A \subset X$, (ii) $A \in SGC(X)$, $A \in PO(X)$ and (iii) $B \in SGC(A)$ together imply, by Lemma 4.3, that $B \in SGC(X)$.

Since $p \in A \subset X$ it follows from the *sg*-regularity of X that there exist U, $V \in SO(X)$, such that $p \in U$, $B \subset V$ and $U \cap V = \phi$. Now U, $V \in SO(X)$, $A \in PO(X) \Rightarrow U \cap A$, $V \cap A \in SO(A)$, by Lemma 4.1. Consequently, for $p \in A$ and $p \notin B \in SGC(A)$ there exist $U \cap A$, $V \cap A \in SO(A)$ such that $p \in U \cap A$, $B \subset V \cap A$ and $(U \cap A) \cap (V \cap A) = A \cap (U \cap V) = \phi$. This indicates *sg*-regularity of the subspace A.

Theorem 4.2. *The semi-homeomorphic image (s.h.C.H.) of a sg-regular space is sg-regular.*

Proof. Let $f: X \to Y$ be a semi-homeomorphism (s.h.C.H.). Let $F \in SGC(Y)$ with $y \notin F$. Since $F \in SGC(Y)$, $scl(F) \subset O$ whenever $F \subset O \in SO(Y)$. The bijectiveness of f gives the existence of a $x \in X$ such that $x = f^{-1}(y)$. Also $f^{-1}[scl(F)] \subset f^{-1}[O]$ whenever $f^{-1}[F] \subset f^{-1}[O]$. This implies $scl(f^{-1}[F]) \subset f^{-1}[O]$. Since f is a semi-homeomorphism, $f^{-1}[O] \in SO(X)$. Thus $f^{-1}[F]$ is sg-closed in X. Since X is sg-regular, for every sg-closed set $f^{-1}[F]$ in X, $x \notin f^{-1}[F]$, there exist two disjoint s.o. sets O_1, O_2 in X such that $x \in O_1$ and $f^{-1}[F] \subset O_2$, i.e. $F \subset f[O_2]$. Also, $O_1 \cap O_2 = \phi$. Now, $f[O_1] \cap f[O_2] = f[O_1 \cap O_2] = f[\phi] = \phi$. The presemi-openness of f yields that $f[O_1]$, $f[O_2] \in SO(Y)$. Thus $y \in f[O_1]$, $F \subset f[O_2]$ and $f[O_1] \cap f[O_2] = \phi$. This ensures that Y is sg-regular.

Corollary 4.1. *sg-regularity is a topological property.*

Proof. Since every semi-topological property is a topological property, the result follows from the Theorem 4.2.

5 - Some basic results of sg-regular spaces

We begin this subsection with the following remark and example.

Remark 5.1. A semi-regular space is not, in general, semi-$T_{\frac{1}{2}}$ as shown by

Example 5.1. The space (X, τ) in Example 3.2 is not semi-$T_{\frac{1}{2}}$. However, we have the following

Theorem 5.1. *A semi-regular space which is also semi-$T_{\frac{1}{2}}$ is sg-regular.*

Proof. Let $F \in SC(X)$ with $x \notin F$. Since X is semi-regular there exist U, $V \in SO(X)$ such that $x \in U$, $F \subset V$ and $U \cap V = \phi$. Since X is semi-$T_{\frac{1}{2}}$, $SC(X) = SGC(X)$. Consequently, $F \in SGC(X)$. Hence X is sg-regular.

Theorem 5.2. *If $x \in X \in SGRS$ with $x \notin A \in SGC(X)$ then there exist U, $V \in SO(X)$ such that $x \in U$, $A \subset V$ and $(scl(U)) \cap (scl(V)) = \phi$.*

Proof. Since $A \in SGC(X)$ and $x \notin A$, the sg-regularity of x gives the existence of two sets W, $U \in SO(X)$ such that $x \in W$, $A \subset U$ and $W \cap U = \phi$. This disjointness of W and U indicates $W \cap (scl(U)) = \phi$. By Theorem 3.1, $SGRS \subset SRS$. Hence, by the semi-regularity of X together with the semi-openness of W there exists $V \in SO(X, x)$ such that $x \in V \subset scl(V) \subset W$. From this it, then, follows that $(scl(U)) \cap (scl(V)) = \phi$.

To continue the study of *sg*-regular space further, the following definition and lemma are required.

Definition 5.1. A topological space (X, τ) is termed semi-generalised normal (briefly *sg*-normal) if for every pair of disjoint sets $A, B \in SGC(X)$ there exist $U, V \in SO(X)$ such that $A \subset U$, $B \subset V$ and $U \cap V = \phi$.

Lemma 5.1. *A space (X, τ) is semi-R_0 iff $\{x\} \in SGC(X)$ for every $x \in X$.*

Proof. Follows from the definitions of the *sg*-closed set and the semi-R_0 space.

Theorem 5.3. *If a space X is sg-normal and semi-R_0, then $X \in SGRS$.*

Proof. Let $A \in SGC(X)$ and $x \notin A$. This implies $x \in X - A \in SGO(X)$. Semi-R_0-ness of X assumes that $\{x\} \in SGC(X)$. So, A and $\{x\}$ are two disjoint *sg*-closed sets. From the *sg*-normality of X, one can, then, infer that there exist U, $V \in SO(X)$ such that $scl(\{x\}) \subset U$, $A \subset V$ and $U \cap V = \phi$. This means $x \in U$, $A \subset V$ and $U \cap V = \phi$. So, X is *sg*-regular. Hence, the theorem.

Lemma 5.2. *A space (X, τ) is extremally disconnected and $A, B \in SO(X)$ then $A \cap B \in SO(X)$.*

Proof. Semi-opennes of A and B yields $A \cap B \subset (Cl(\text{Int}(A)) \cap Cl(\text{Int}(B)))$. Extremally disconnectedness of X implies openness of $Cl(\text{Int}(B))$. Hence $A \cap B \subset (Cl(\text{Int}(A)) \cap (Cl(\text{Int}(B))) \subset Cl(Cl(\text{Int}(B) \cap \text{Int}(A))) = Cl(\text{Int}(A \cap B))$. So, $A \cap B \in SO(X)$.

Remark 5.2. The intersection of any finite family of s.o. sets in an extremally disconnected space is a s.o. set.

Theorem 5.4. *If (X, τ) is sg-regular and extremally disconnected then for every pair of disjoint sets A, B where A is semi-compact and $B \in SGC(X)$, there exist $U, V \in SO(X)$ such that $A \subset U$, $B \subset U$ and $U \cap V = \phi$.*

Proof. Let $x \in A$. Then $A \cap B = \phi \Rightarrow x \notin B \in SGC(X)$. From the *sg*-regularity of X, then, it follows that there exist $U_x, V_x \in SO(X)$ such that $x \in U_x$, $B \subset V_x$ and $U_x \cap V_x = \phi$. The family $\{U_x : X \in \Lambda\}$ is clearly a cover of A by s.o. sets. The semi-compactness of A then assures the existence of a finite number of points x_1, x_2, x_3, $\dots$, x_n in A such that $A \subset \bigcup_{i=1}^{n} U_{x_i}$. Let $U = \bigcup_{i=1}^{n} U_{x_i}$. Then, by Theorem 2.2, $U \in SO(X)$. Now consider the corresponding $V_{x_i} \in SO(X)$ such that $B \subset V_{x_i}$ where $i = 1, 2, 3, \dots, n$. Extremally disconnectness of X guarantees, by Remark 5.2, that $V = \bigcap_{i=1}^{n} V_{x_i} \in SO(X)$ with $B \subset V$. Thus, we obtain, U, $V \in SO(X)$ such that $A \subset U$, $B \subset V$ and $U \cap V = \phi$.

Theorem 5.5. *Let* $\{X_\alpha, \tau_\alpha): \alpha \in \Lambda\}$ *be a family of spaces where* $\{X_\alpha: \alpha \in \Lambda\}$ *is a family of pairwise disjoint sets. Let* (X, τ) *be the topological sum of* $\{(X_\alpha, \tau_\alpha): \alpha \in \Lambda\}$. *Then* (X, τ) *is sg-regular if* (X_α, τ_α) *is sg-regular for each* $\alpha \in \Lambda$.

Proof. Suppose $F \in SGC(X)$ and $x \notin F$. Since X is a topological sum, each X_α is closed in X and hence $X_\alpha \in SGC(X)$ for each $\alpha \in \Lambda$. It is clear from Dontchev's Theorem that $F_\alpha \in SGC(X)$ where $F_\alpha = F \cap X_\alpha$. Again the openness (hence preopennes) of X_α in X together with the fact that $F_\alpha \subset X_\alpha \subset X$ yields, by Lemma 4.3, that $F_\alpha \in SGC(X_\alpha)$. Also $x \notin F_\alpha$. By *sg*-regularity of X_α there exist U_α, $V_\alpha \in SO(X_\alpha)$ such that $x \in U_\alpha$, $F_\alpha \subset V_\alpha$ and $U_\alpha \cap V_\alpha = \phi$. For $\alpha \neq \lambda$, let $V = \bigcup_{\alpha \neq \lambda} V_\lambda$ where $V_\lambda = X_\lambda$. By Theorem 2.2, $V \in SO(X)$. On the other hand, $U_\alpha \in SO(X_\alpha)$, $X_\alpha \in SO(X)$ and $U_\alpha \subset X_\alpha \subset X$ together give, by Theorem 2.3, $U_\alpha \in SO(X)$. Obviously, $F \subset V \cup X_\alpha$ so that $F = F \cap (V \cup X_\alpha) \subset V \cup F_\alpha \subset V \cup V_\alpha = \widehat{V}_\alpha$, say. Again from the mutual disjointness of X_α's one infers that $U_\alpha \cap V = \phi$ whence $U_\alpha \cap \widehat{V}_\alpha = U_\alpha \cap (V \cup V_\alpha) = \phi$. Thus, for $x \notin F \in SGC(X)$ there exist U_α, $\widehat{V}_\alpha \in SO(X)$ such that $x \in U_\alpha$, $F \subset \widehat{V}_\alpha$ and $U_\alpha \cap \widehat{V}_\alpha = \phi$. This guarantees the *sg*-regularity of X. Hence the theorem.

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Abstract

In this paper a new separation axiom termed sg-regularity has been introduced, characterised and its basic properties have been obtained. sg-regularity of topological sum has also been studied. Besides this, it exhibits the role of sg-closed sets in topology.

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