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Exponential stability of a viscoelastic plate with thermal memory (**)

1 - Introduction

In this paper we investigate the asymptotic behavior of solutions of a problem describing temperature and vertical displacement evolution in a homogeneous, thermally isotropic, Kirchhoff plate composed of material with linear memory and subject to thermal deformations. In addition, a non–Fourier constitutive law for the heat flux is considered here. The resulting model has been derived in the framework of the well–established theory of heat flow with memory due to Gurtin and Pipkin [**5**] and will appear in [**3**]. In the sequel we sketch the modelling procedure.

We assume that the plate occupies a fixed bounded domain $\Omega \subset \mathbb{R}^2$, with Lipschitz boundary $\Gamma = \Gamma_0 \cup \Gamma_1$ and is rigidly clamped along Γ_0 and simply supported along Γ_1. In addition we suppose that $\overline{\Gamma}_0 \cap \overline{\Gamma}_1 \neq \emptyset$.

The material composing the plate is *isotropic* (mechanically and thermally) and *viscoelastic*, so that its *stress-strain* law is given by

$$\boldsymbol{S}(\boldsymbol{x}, t) = \mathbb{G} * \boldsymbol{E}_t(\boldsymbol{x}, t) - \alpha_0 \theta(\boldsymbol{x}, t) \boldsymbol{I} \tag{1.1}$$

where $\boldsymbol{S}$ and $\boldsymbol{E}$ denote the *stress* and *strain tensors*, respectively, $*$ denotes con-

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volution, $\dot{} = \dfrac{\partial}{\partial t}$, and

$$\mathbb{G}(\tau) = \lambda(\tau)\, \boldsymbol{I} \otimes \boldsymbol{I} + 2\sigma(\tau)\, \mathbb{I}\,, \qquad \text{for } \tau \geqslant 0$$

is a fourth order tensor involving two independent *relaxation functions*

$$\lambda,\, \sigma : \mathbb{R}^+ \to \mathbb{R}\,.$$

The last term in (1.1) represents the *thermal stress* and α_0 is a positive constant. Moreover, let $\boldsymbol{q}(\boldsymbol{x}, t) : \Omega \times \mathbb{R} \to \mathbb{R}^3$ be the *heat flux vector* in the plate. According to the linearized Gurtin-Pipkin heat law for a thermally isotropic body, we assume

$$\boldsymbol{q}(\boldsymbol{x}, t) = -\,\kappa * \nabla \tilde{\theta}(\boldsymbol{x}, t) \tag{1.2}$$

where θ denotes *absolute temperature*, $\tilde{\theta}(\boldsymbol{x}, t-s) = \int_0^s \theta(\boldsymbol{x}, t-\tau)\, d\tau$ is the *summed temperature history* and $\kappa : \mathbb{R}^+ \to \mathbb{R}$ is the *heat flux memory kernel*. Unfortunately, we are not allowed here to use the corresponding linearized expression of the internal energy and its balance equation. Indeed, the theory of Gurtin and Pipkin only applies to rigid heat conductors, so that we must resort to some generalization if small deformations are taken into account. Therefore, we take advantage of the thermodynamically consistent theory of linear thermoviscoelasticity proposed in [**9**]. There, the usual energy balance equation is replaced by

$$\varrho_0 h(\boldsymbol{x}, t) = -\nabla \cdot \boldsymbol{q}(\boldsymbol{x}, t) + \varrho_0 r(\boldsymbol{x}, t) \tag{1.3}$$

where h is the *thermal power*, which denotes the rate of heat absorption per unit of volume, $\varrho_0 > 0$ is the *density* of the medium, and r is the *external heat supply per unit of mass*. Neglecting any hereditary contribution to mechanical dissipation, h is described by the following linearized constitutive equation (see [**9**]):

$$h(\boldsymbol{x}, t) = \frac{\theta_0}{\varrho_0}\left[\boldsymbol{B} : \boldsymbol{E}_t(\boldsymbol{x}, t) + \frac{\varrho_0 c}{\theta_0}\vartheta_t(\boldsymbol{x}, t) + a * \vartheta_t(\boldsymbol{x}, t)\right] \tag{1.4}$$

where $\boldsymbol{B}$ is a symmetric second order tensor, $a : \mathbb{R}^+ \to \mathbb{R}$ is the *energy memory kernel*, $c > 0$ is the *specific heat* of body, $\vartheta = \theta - \theta_0$ and θ_0 is the *reference temperature*. Dealing with rigid conductors ($\boldsymbol{B} \equiv \boldsymbol{0}$), h reduces to time derivative of the Gurtin–Pipkin's integral energy after integrating by parts, provided that $\lim_{s \to \infty} a(s) = 0$. This procedure is consistent with respect to the *time reversal property* and thermodynamic principles (as shown in [**3**]).

Putting (1.2) and (1.4) into the energy balance equation (1.3) with zero *exter-*

nal heat supply, and paralleleling the procedures of [7] and [8] in the framework of hereditary materials, we obtain the following model of a linear viscoelastic plate with thermal memory (see [3] for more details)

$$(1.5)\qquad \begin{cases} w_{tt}(t) + g(0)\,\Delta^2 w(t) + \Delta^2 \int_0^\infty g'(s)\, w^t(s)\, ds + \alpha\,\Delta\vartheta(t) = 0 \\ \vartheta_t(t) + \beta\vartheta(t) + \Delta \int_0^\infty \mu'(s)\, \tilde{\vartheta}^t(s)\, ds - \int_0^\infty \gamma'(s)\, \tilde{\vartheta}^t(s)\, ds - \alpha\,\Delta w_t(t) = 0 \end{cases}$$

where the dependence on $\boldsymbol{x} \in \Omega$ is understood. In (1.5) w is the *bending component* of the plate displacement, $\beta = \dfrac{\theta_0 a(0)}{\varrho_0 c}$ is a positive constant, Δ is the bidimensional Laplace operator and

$$w^t(s) = w(t-s)\,, \qquad \tilde{\vartheta}^t(s) = \int_0^s \vartheta(t-\tau)\, d\tau\,.$$

Memory kernels g, μ and γ are related to λ, σ, κ and a by means of suitable algebric expressions of their Laplace transforms. Let $\varphi(t)$ be a function taking values in a Hilbert space, and let denote by $\widehat{\varphi}$ its Laplace transform, namely

$$\widehat{\varphi}(s) = \int_0^\infty e^{-st}\varphi(t)\, dt\,.$$

Then, introducing the *viscoelastic Poisson's ratio* ν and *viscoelastic Young's modulus* E so that their Laplace transforms are respectively defined by

$$\widehat{\nu}(s) = \frac{\widehat{\lambda}(s)}{2s[\widehat{\lambda}(s) + \widehat{\sigma}(s)]}\,, \qquad \widehat{E}(s) = \frac{\widehat{\sigma}(s)[3\,\widehat{\lambda}(s) + 2\,\widehat{\sigma}(s)]}{\widehat{\lambda}(s) + \widehat{\sigma}(s)}\,,$$

we have (see [3])

$$\widehat{g}(s) = \frac{\widehat{E}(s)\, d^3}{12(1 - s^2\widehat{\nu}^2(s))}$$

where d is the uniform thickness d of the thin plate. On the other hand we assume (see [3])

$$\mu(s) = \frac{\kappa(s)}{\varrho_0 c}, \qquad \gamma(s) = \frac{\theta_0 a'(s)}{\varrho_0 c} + \frac{12}{d^2}\mu(s), \qquad \alpha = \frac{\theta_0 \beta(3\lambda_0 + 2\sigma_0)}{\varrho_0 c},$$

where $\lambda_0 = \lambda(0)$ and $\sigma_0 = \sigma(0)$.

The structural boundary conditions for the clamped–supported plate are as follows (see [3], [7]):

$$w(t) = \frac{\partial w(t)}{\partial n} = 0 \qquad \text{on } \Gamma_0 \times (0, +\infty) \tag{1.6}$$

$$w(t) = \mathcal{B}[(I+G)\, w](t) + \alpha\vartheta(t) = 0 \quad \text{on } \Gamma_1 \times (0, +\infty) \tag{1.7}$$

where $n = (n_1, n_2)$ is the unit outward normal vector,

$$\mathcal{B}\varphi = \Delta\varphi + (1 - \nu_0)\, B_1 \varphi$$

is a boundary operator defined by

$$B_1 \varphi = 2 n_1 n_2 \frac{\partial^2 \varphi}{\partial x \partial y} - n_1^2 \frac{\partial^2 \varphi}{\partial y^2} - n_2^2 \frac{\partial^2 \varphi}{\partial x^2}$$

and

$$Gw = g' * w .$$

The temperature boundary condition is (see [3])

$$[\lambda_1 F + \lambda_2 I]\, \vartheta(t) = 0 \qquad \text{on } \Gamma \times (0, +\infty) \tag{1.8}$$

where $\lambda_1, \lambda_2 \geq 0$, $\lambda_1 + \lambda_2 \neq 0$ and

$$F\vartheta(t) = \mu * \frac{\partial \vartheta(t)}{\partial n} = \int_0^\infty \mu(s) \frac{\partial \vartheta(t-s)}{\partial n}\, ds .$$

Remark 1.1. We observe that $\lambda_1 = 0$ corresponds to zero temperature at the boundary. The case $\lambda_1 > 0$ leads to a variant of Newton's cooling law, namely

$$F\vartheta(t) = -\lambda\vartheta(t), \qquad \text{with } \lambda = \frac{\lambda_2}{\lambda_1}.$$

In particular if $\lambda_2 = 0$ we have zero heat flux at the boundary.

The initial state of the plate is given by the initial values of displacement, velocity and temperature (w_0, v_0, ϑ_0) and, because of the memory terms, by the initial histories of displacement and temperature (w^0, ϑ^0)

$$
\begin{aligned}
&w(\boldsymbol{x}, 0) = w_0(\boldsymbol{x}),\ w_t(\boldsymbol{x}, 0) = v_0(\boldsymbol{x}),\ \vartheta(\boldsymbol{x}, 0) = \vartheta_0(\boldsymbol{x}) \\
&w(\boldsymbol{x}, -s) = w^0(\boldsymbol{x}, s),\ \vartheta(\boldsymbol{x}, -s) = \vartheta^0(\boldsymbol{x}, s), \qquad \forall s \in \mathbb{R}^+ .
\end{aligned}
\tag{1.9}
$$

Concerning the constitutive *relaxation functions* g, μ and γ in (1.5), we assume the following set of hypotheses.

(h1) $$g, \mu, \gamma \in C^2(0, \infty) \cap C[0, \infty), \qquad g, \mu, \gamma > 0, \qquad \text{on } \mathbb{R}^+$$

(h2) $$g(0) = 1, \qquad g(\infty) = g_\infty > 0, \qquad \mu(\infty) = \gamma(\infty) = 0$$

(h3) $$g', \gamma', \mu' < 0, \qquad g'', \gamma'', \mu'' > 0 \qquad \text{on } \mathbb{R}^+ .$$

In addition, if we expect to achieve exponential decay of the energy, we must assume that g', μ' and γ' decay exponentially as $s \to \infty$, namely there exist three positive constants δ_1, δ_2 and δ_3 such that

(h4) $$g''(s) + \delta_1 g'(s) \leqslant 0 , \qquad \text{for } s \in \mathbb{R}^+$$

(h5) $$\mu''(s) + \delta_2 \mu'(s) \leqslant 0 , \qquad \text{for } s \in \mathbb{R}^+$$

(h6) $$\gamma''(s) + \delta_3 \gamma'(s) \leqslant 0 , \qquad \text{for } s \in \mathbb{R}^+ .$$

In recent years, many efforts are devoted to studying thermoelastic Kirchhoff plate models. Several authors studied the problem

$$
\begin{cases}
w_{tt} - \gamma \Delta w_{tt} + \Delta^2 w + \alpha \Delta \vartheta = 0 \\
\beta \vartheta_t - \eta \Delta \vartheta + \sigma \vartheta - \alpha \Delta w_t = 0
\end{cases}
$$

taken from J. Lagnese's monograph [**8**]. When the rotatory inertia is neglected in the plate equation, i.e. $\gamma = 0$, J. Kim in [**6**] proved the uniform stability of the thermoelastic system with the clamped boundary conditions $w = \dfrac{\partial w}{\partial n} = \vartheta = 0$. J. E. Muñoz Rivera and R. Racke [**13**] analysed the coupled equation with the hinged boundary conditions $w = \Delta w = \vartheta = 0$. K. Liu and Z. Liu [**11**] developed an abstract framework for analysis of linear thermoelastic systems and, in particular, for homogeneous and nonhomogeneous Kirchhoff plates with different sets of boundary conditions. Z. Liu and S. Zheng [**12**] established the exponential stability of the semigroup associated with the Kirchhoff plate under thermal or viscoelastic damping. G. Avalos and I. Lasiecka [**1**] determined the uniform stability of the thermoelastic plate with no added dissipative mechanism on the boundary.

Our purpose is to extend previously quoted results to viscoelastic plates with thermal memory by paralleling a procedure successfully applied to viscoelastic bars [**2**]. In particular, we show the exponential decay of energy of the linear Kirchhoff viscoelastic plate with thermal memory subject to fixed boundary conditions. In spite of the presence of a convolution term, the original problem is trasformed into an autonomous system by suitable choice of variables. As a consequence, linear semigroup theory is used and the exponential stability is proved for a class of memory functions including weakly singular kernels which decay exponentially for large time.

2 - Functional setting and notation

Let $\Omega \subset \mathbb{R}^2$. With usual notation, we introduce the space L^2 acting on Ω. Hereafter, $\langle \cdot , \cdot \rangle$ denotes the L^2 inner product, and $\|\cdot\|$ denotes the L^2 norm. Let

$$H^k_{\Gamma_j} = \left\{ \varphi \in H^k(\Omega) \,\middle|\, \frac{\partial^i \varphi}{\partial n^i} = 0 \ \text{ on } \Gamma_j, \ \text{ for } i = 0, \ldots, k-1 \right\}, \qquad j = 0, 1 .$$

Moreover, using Green's formula, we have

$$\tag{2.1} \begin{aligned} \int_\Omega (\Delta^2 \varphi_1)\, \varphi_2 \, d\Omega &= a(\varphi_1, \varphi_2) \\ &+ \int_\Gamma \left[\frac{\partial \Delta \varphi_1}{\partial n} + (1-\nu_0) \frac{\partial B_2 \varphi_1}{\partial \tau} \right] \varphi_2 \, d\Gamma - \int_\Gamma [\Delta \varphi_1 + (1-\nu_0) B_1 \varphi_1] \frac{\partial \varphi_2}{\partial n} \, d\Gamma \end{aligned}$$

where $\tau = (-n_2, n_1)$ is the unit tangent vector. The bilinear form a is given by

$$\begin{aligned} a(\varphi_1, \varphi_2) = \int_\Omega [& \varphi_{1xx} \varphi_{2xx} + \varphi_{1yy} \varphi_{2yy} + \nu_0 (\varphi_{1xx} \varphi_{2yy} + \varphi_{1yy} \varphi_{2xx}) \\ & + 2(1-\nu_0)\, \varphi_{1xy} \varphi_{2xy}] \, d\Omega \end{aligned}$$

and B_2 is a boundary operator defined by

$$B_2 \varphi_1 = (n_1^2 - n_2^2)\, \varphi_{1xy} + n_1 n_2 (\varphi_{1yy} - \varphi_{1xx}) .$$

We introduce the following Hilbert spaces

$$M = \left\{ \varphi : \mathbb{R}^+ \to H^2_{\Gamma_0} \cap H^1_{\Gamma_1} \,\middle|\, \int_0^\infty |g'(s)|\, a(\varphi(s))\, ds < +\infty \right\}$$

$$N = \left\{ \varphi : \mathbb{R}^+ \to H^1 \,\middle|\, \int_0^\infty [\,|\mu'(s)|\, \|\nabla\varphi(s)\|^2 + |\gamma'(s)|\, \|\varphi(s)\|^2]\, ds < +\infty \right\}$$

respectively endowed with the inner products

$$\langle \varphi_1, \varphi_2 \rangle_g = \int_0^\infty |g'(s)|\, a(\varphi_1(s), \varphi_2(s))\, ds\,,$$

$$\langle \varphi_1, \varphi_2 \rangle_{\mu, \gamma} = \int_0^\infty [\,|\mu'(s)|\, \langle \nabla\varphi_1(s), \nabla\varphi_2(s)\rangle\, ds + |\gamma'(s)|\, \langle \varphi_1(s), \varphi_2(s)\rangle]\, ds\,.$$

In order to rewrite problem (1.5)–(1.9) in a history space setting, we introduce the Hilbert space

$$Z := U \times V \times \Theta \times M \times N$$

where $U := H^2_{\Gamma_0} \cap H^1_{\Gamma_1}$, $V := L^2$, $\Theta := L^2$ and whose inner product is given by

$$\langle z_1, z_2 \rangle_Z = \langle w_1, w_2 \rangle_U + \langle v_1, v_2 \rangle_V + \langle \vartheta_1, \vartheta_2 \rangle_\Theta + \langle \psi_1, \psi_2 \rangle_M + \langle \eta_1, \eta_2 \rangle_N$$

$$= g_\infty\, a(w_1, w_2) + \langle v_1, v_2 \rangle + \langle \vartheta_1, \vartheta_2 \rangle + \int_0^\infty |g'(s)|\, a(\psi_1, \psi_2)\, ds$$

$$+ \int_0^\infty [\,|\mu'(s)|\, \langle \nabla\eta_1, \nabla\eta_2 \rangle + |\gamma'(s)|\, \langle \eta_1, \eta_2 \rangle]\, ds$$

with $z_i = (w_i, v_i, \vartheta_i, h_i, \eta_i)^T$, $i = 1, 2$.

Let

$$a(\varphi) = a(\varphi, \varphi) = \int_\Omega [\varphi^2_{xx} + \varphi^2_{yy} + 2\nu_0 \varphi_{xx} \varphi_{yy} + 2(\nu_0 - 1)\, \varphi^2_{xy}]\, d\Omega$$

according to [**16**].

Remark 2.1. We can see that

$$a(\varphi) \geq C\|\varphi\|_{H^2}, \qquad \forall \varphi \in H^2_{\Gamma_0} \cap H^1_{\Gamma_1} \tag{2.2}$$

for some constant $C > 0$.

We conclude this introductory part with some basic facts about semigroup of operators. For a detailed exposition of the subject the reader is referred to [**14**] and [**15**]. In the sequel of this section, let H denote a real Hilbert space endowed with the scalar product $\langle \cdot, \cdot \rangle$ and norm $\|\cdot\|$. Since no confusion should occur, we denote again by $\|\cdot\|$ the norm of a bounded operator on H.

Theorem 2.1 (Lumer-Phillips). *Let $\mathcal{L}$ be a linear operator with dense domain $D(\mathcal{L})$ in H. If $\mathcal{L}$ is dissipative and there is a $\lambda_0 > 0$ such that the range, $R(\lambda_0 I - \mathcal{L})$, of $\lambda_0 I - \mathcal{L}$ is H, then $\mathcal{L}$ is the infinitesimal generator of a C_0-semigroup of contractions on H.*

For later convenience we recall that a linear C_0-semigroup $T(t)$ of contractions is said to be *exponentially stable* if there exist two constants $M \geq 1$ and $\beta > 0$ such that

$$\|T(t)\, z_0\|_H \leq M e^{-\beta t}\|z_0\|_H, \qquad \forall z_0 \in H, \ \ \forall t > 0\,. \tag{2.3}$$

We recall that the *complexification* of H is the complex Hilbert space $H_{\mathbb{C}}$, defined by

$$H_{\mathbb{C}} = \{z \,|\, z = x + iy, \quad x, y \in H\},$$

endowed with the inner product

$$\langle x_1 + iy_1, x_2 + iy_2\rangle_{\mathbb{C}} = \langle x_1, x_2\rangle + \langle y_1, y_2\rangle + i\langle y_1, x_2\rangle - i\langle x_1, y_2\rangle\,.$$

Analogously, the complexification $\mathcal{L}_{\mathbb{C}}$ of $\mathcal{L}$ is the linear operator on $H_{\mathbb{C}}$ with domain

$$D(\mathcal{L}_{\mathbb{C}}) = \{z \,|\, z = x + iy, \quad x, y \in D(\mathcal{L})\}$$

defined by

$$\mathcal{L}_{\mathbb{C}}(x + iy) = \mathcal{L}x + i\mathcal{L}y$$

and the corresponding semigroup $S(t)$ on $\mathcal{H}_{\mathbb{C}}$ is defined by

$$S(t)(x + iy) = T(t)\, x + iT(t)\, y\,.$$

In order to prove the exponential stability of the semigroup $T(t)$ generated by $\mathcal{L}$ we shall use a crucial property of its spectrum. The basic result, which traces back to Prüss [**15**], appears in the relevant literature under some equivalent statements. Here, we recall the following (see [**4**])

Lemma 2.1. *Let $T(t)$ be a contraction semigroup on a real Hilbert space H, let $\mathcal{L}$ be its infinitesimal generator. If the operator $i\beta I - \mathcal{L}_{\mathbb{C}}$ is uniformly bounded below as $\beta \in \mathbb{R}$, that is, if there exists $\sigma > 0$ such that*

$$\inf_{\beta \in \mathbb{R}} \|(i\beta I - \mathcal{L}_{\mathbb{C}})\, z\|_{\mathbb{C}} \geq \sigma \|z\|_{\mathbb{C}} \qquad \forall z \in D(\mathcal{L}_{\mathbb{C}}) \subset H_{\mathbb{C}} \tag{2.4}$$

then $T(t)$ is exponentially stable.

3 - Existence and uniqueness

After introducing the new variables

$$v(t) = w_t(t) \tag{3.1}$$

$$\psi^t(s) = w(t) - w(t-s) \tag{3.2}$$

$$\eta^t(s) = \int_0^s \vartheta(t-\tau)\, d\tau \tag{3.3}$$

and setting

$$z = (w,\, v,\, \vartheta,\, \psi,\, \eta)^T,$$

system (1.5) can be written as an abstract evolution equation

$$z_t(t) = \mathcal{A} z(t) \tag{3.4}$$

on the Hilbert space Z where the operator $\mathcal{A}$ in (3.4) is given by

$$\mathcal{A}\begin{pmatrix} w \\ v \\ \vartheta \\ \psi \\ \eta \end{pmatrix} = \begin{pmatrix} v \\ -g_\infty \Delta^2 w + \int_0^\infty g'(s)\, \Delta^2 \psi(s)\, ds - \alpha\Delta\vartheta \\ \alpha\Delta v - \beta\vartheta - \int_0^\infty \mu'(s)\, \Delta\eta(s)\, ds + \int_0^\infty \gamma'(s)\, \eta(s)\, ds \\ v - \psi_s \\ \vartheta - \eta_s \end{pmatrix}$$

and its domain is defined as

$$D(\mathcal{A}) = \left\{ z \in Z \;\middle|\; \begin{array}{l} w \in H^4 \cap H^2_{\Gamma_0} \cap H^1_{\Gamma_1}, \\ v \in H^2_{\Gamma_0} \cap H^1_{\Gamma_1},\ \vartheta \in H^2, \\ \int_0^\infty g'(s)\, \Delta^2 \psi(s)\, ds \in L^2, \\ \int_0^\infty \mu'(s)\, \Delta\eta(s)\, ds - \int_0^\infty \gamma'(s)\, \eta(s)\, ds \in L^2, \\ \psi(s) \in H^1(\mathbb{R}^+;\ |g'|;\ H^2_{\Gamma_0} \cap H^1_{\Gamma_1}),\ \psi(0) = 0, \\ \eta(s) \in H^1(\mathbb{R}^+;\ |\mu'|;\ |\gamma'|;\ H^1),\ \eta(0) = 0 \\ w \text{ e } \vartheta \text{ satisfy (1.6)-(1.8)} \end{array} \right\}.$$

By (3.2) and (h2), the structural boundary conditions (1.6)-(1.7) become

$$w = \frac{\partial w}{\partial n} = 0 \qquad \text{on } \Gamma_0 \times (0, +\infty) \tag{3.6}$$

$$w = \mathcal{B}\left(g_\infty w - \int_0^\infty g'(s)\, \psi(s)\, ds \right) + \alpha\vartheta = 0 \qquad \text{on } \Gamma_1 \times (0, +\infty) \tag{3.7}$$

and the temperature boundary condition (1.8) reduces to

$$\lambda_1 \int_0^\infty \mu'(s)\, \frac{\partial \eta(s)}{\partial n}\, ds - \lambda_2 \vartheta = 0 \qquad \text{on } \Gamma \times (0, +\infty). \tag{3.8}$$

Remark 3.1. Taking into account (3.8), it follows that

$$\int_{\Gamma}\left[\int_0^\infty \mu'(s)\,\frac{\partial\eta(s)}{\partial n}\,ds\right]\vartheta\,d\Gamma = \begin{cases} 0 & \text{if } \lambda_1 = 0 \\ \lambda\int_{\Gamma}\vartheta^2\,d\Gamma & \text{if } \lambda_1 > 0 \end{cases} \tag{3.9}$$

where $\lambda = \dfrac{\lambda_2}{\lambda_1} \geqslant 0$.
According to (1.9), initial conditions can be written as

$$z(0) = z_0, \qquad z_0 \in Z \tag{3.10}$$

where $z_0 = (w_0, v_0, \vartheta_0, \psi^0, \eta^0)$ and

$$\psi^0(s) = w_0 - w^0(s), \qquad \eta^0(s) = \int_{-s}^{0}\vartheta(\tau)\,d\tau = \int_0^s \vartheta^0(\tau)\,d\tau\,.$$

The energy of the thermoviscoelastic plate is represented by

$$E(t) = \frac{1}{2}\left\{g_\infty\,a(w(t)) + \|v(t)\|^2 + \|\vartheta(t)\|^2 + \int_0^\infty |g'(s)|\,a(\psi)\,ds\right.$$

$$\left.+\int_0^\infty[\,|\mu'(s)|\,\|\nabla\eta\|^2 + |\gamma'(s)|\,\|\eta\|^2]\,ds\right\} = \frac{1}{2}\,\|z(t)\|_Z^2.$$

In the sequel we shall prove the well-posedness of the initial boundary value problem (3.4)-(3.10). In view of Theorem 2.1 we prove the following Theorem.

Theorem 3.1. *If the memory kernels satisfy* (h1)-(h3), *then* $\mathcal{A}$ *is the infinitesimal generator of a* C_0*-semigroup of contraction on* Z.

Proof. First, we prove that $\mathcal{A}$ is dissipative. For every $z \in D(\mathcal{A})$ we have

$$\langle\mathcal{A}z, z\rangle_Z = \langle v, w\rangle_U + \left\langle -g_\infty\Delta^2 w + \int_0^\infty g'(s)\,\Delta^2\psi(s)\,ds - \alpha\Delta\vartheta,\, v\right\rangle_V$$

$$+\left\langle \alpha\Delta v - \beta\vartheta - \int_0^\infty \mu'(s)\,\Delta\eta(s)\,ds + \int_0^\infty \gamma'(s)\,\eta(s)\,ds,\,\vartheta\right\rangle_\Theta + \langle v - \psi_s, \psi\rangle_M$$

$$+\langle\vartheta - \eta_s, \eta\rangle_N = -\beta\|\vartheta\|^2 - \int_\Gamma\left[\int_0^\infty \mu'(s)\,\frac{\partial\eta(s)}{\partial n}\,ds\right]\vartheta\,d\Gamma - \langle\psi_s, \psi\rangle_M - \langle\eta_s, \eta\rangle_N.$$

Integrating by parts, in view of (h3), it follows that

$$
\begin{aligned}
&\langle \psi_s, \psi \rangle_M = \frac{1}{2} \int_0^\infty g''(s)\, a(\psi(s))\, ds \geq 0 \\
(3.11) \qquad &\langle \eta_s, \eta \rangle_N = \frac{1}{2} \int_0^\infty [\mu''(s) \|\nabla \eta(s)\|^2 + \gamma''(s) \|\eta(s)\|^2]\, ds \geq 0\,.
\end{aligned}
$$

The above calculation is obtained formally taking product in M and N and can be made rigorous with the use of mollifiers (see [4]).

Remark 3.2. Because of (3.9), we have either $\vartheta = 0$ or $\int_0^\infty \mu'(s) \frac{\partial \eta}{\partial n}\, ds = \lambda \vartheta$, for $\lambda \geq 0$, so that

$$
\int_\Gamma \left(\int_0^\infty \mu'(s) \frac{\partial \eta}{\partial n}\, ds \right) \vartheta\, d\Gamma \geq 0\,.
$$

Thus we obtain

$$
\langle \mathcal{A} z, z \rangle_Z \leq 0
$$

so proving the dissipativeness of $\mathcal{A}$.

Now we prove that the operator $I - \mathcal{A}$ is surjective. In order to determine the range of $I - \mathcal{A}$, we consider the system

$$
(I - \mathcal{A})\, z = \widehat{z}\,,
$$

where $\widehat{z} = (\widehat{w}, \widehat{v}, \widehat{\vartheta}, \widehat{\psi}, \widehat{\eta}) \in Z$, namely

$$
(3.12) \qquad \begin{cases}
w - v = \widehat{w} \\
v + g_\infty \Delta^2 w - \int_0^\infty g'(s)\, \Delta^2 \psi(s)\, ds + \alpha \Delta \vartheta = \widehat{v} \\
\vartheta + \beta \vartheta - \alpha \Delta v + \int_0^\infty \mu'(s)\, \Delta \eta(s)\, ds - \int_0^\infty \gamma'(s)\, \eta(s)\, ds = \widehat{\vartheta} \\
\psi - v + \psi_s = \widehat{\psi} \\
\eta - \vartheta + \eta_s = \widehat{\eta}\,.
\end{cases}
$$

Integrating $(3.12)_4$ and $(3.12)_5$ we obtain

$$
\begin{aligned}
\psi(\cdot, s) &= v(\cdot)(1-e^{-s}) + \int_0^s e^{\tau - s}\widehat{\psi}(\cdot, \tau)\, d\tau \\
\eta(\cdot, s) &= \vartheta(\cdot)(1-e^{-s}) + \int_0^s e^{\tau - s}\widehat{\eta}(\cdot, \tau)\, d\tau \, .
\end{aligned}
\tag{3.13}
$$

Substituting v and ψ from $(3.12)_1$ and $(3.13)_1$ into $(3.12)_2$ we obtain

$$
w + c_g \Delta^2 w + \alpha \Delta\vartheta = \widehat{v} + \widehat{w} + \int_0^\infty g'(s)\, \Delta^2 \left[(e^{-s} - 1)\, \widehat{w} + \int_0^s e^{\tau - s} \widehat{\psi}\, d\tau \right] ds \, , \tag{3.14}
$$

and substituting η and v from $(3.13)_2$ and $(3.12)_1$ into $(3.12)_3$ yields

$$
\begin{aligned}
& c_\gamma \vartheta - c_\mu \Delta\vartheta - \alpha \Delta w \\
& = \widehat{\vartheta} - \alpha \Delta \widehat{w} - \int_0^\infty \mu'(s) \int_0^s e^{\tau - s} \Delta \widehat{\eta}\, d\tau\, ds + \int_0^\infty \gamma'(s) \int_0^s e^{\tau - s} \widehat{\eta}\, d\tau\, ds \, ,
\end{aligned}
\tag{3.15}
$$

where

$$
c_g = g_\infty - \int_0^\infty g'(s)(1-e^{-s})\, ds
$$

$$
c_\gamma = 1 + \beta - \int_0^\infty \gamma'(s)(1-e^{-s})\, ds
$$

$$
c_\mu = - \int_0^\infty \mu'(s)(1-e^{-s})\, ds \, .
$$

All these constants are positive by virtue of (h1)-(h3). From (3.8) we obtain

$$
\lambda_1 \left\{ \int_0^\infty \mu'(s)\, \frac{\partial \vartheta(t)}{\partial n}\, (1-e^{-s})\, ds + \int_0^\infty \mu'(s) \int_0^s e^{\tau - s}\, \frac{\partial \widehat{\eta}^t(\tau)}{\partial n}\, d\tau \right\} = \lambda_2\, \vartheta(t) \, .
$$

Moreover, it can be shown that the right-hand sides of (3.14)-(3.15) are in H^{-1}. Multiplying equations (3.14) and (3.15) by $\widetilde{w} \in H^2_{\Gamma_0} \cap H^1_{\Gamma_1}$ and $\widetilde{\vartheta} \in H^1$, respectively, integrating both equations on Ω and considering the structural and temperature boundary conditions (3.6)-(3.8), we obtain

$$
\begin{aligned}
&\int_\Omega w\widetilde{w}\, d\Omega + c_\gamma \int_\Omega \vartheta\widetilde{\vartheta}\, d\Omega + c_\mu \int_\Omega \nabla\vartheta\cdot\nabla\widetilde{\vartheta}\, d\Omega \\
&+ c_\mu \lambda \int_\Gamma \vartheta\widetilde{\vartheta}\, d\Gamma + c_g\, a(w, \widetilde{w}) + \alpha \int_\Omega [\vartheta(\Delta\widetilde{w}) - (\Delta w)\,\widetilde{\vartheta}]\, d\Omega \\
&= \int_\Omega \widehat{w}\widetilde{w}\, d\Omega + \int_\Omega \widehat{v}\widetilde{w}\, d\Omega - \int_0^\infty g'(s)(1-e^{-s})\, a(\widehat{w}, \widetilde{w})\, ds \qquad (3.16)\\
&+ \int_0^\infty g'(s)\left[\int_0^s e^{\tau - s} a(\widehat{\psi}, \widetilde{w})\, d\tau\right] ds \\
&+ \beta \int_\Omega \widehat{\vartheta}\widetilde{\vartheta}\, d\Omega - \int_0^\infty \left[\int_0^s e^{\tau - s} \int_\Omega (\mu'(s)\,\Delta\widehat{\eta}(\tau) - \gamma'(s)\,\widehat{\eta}(\tau))\,\widetilde{\vartheta}\, d\Omega\, d\tau\right] ds\,.
\end{aligned}
$$

By the Lax-Milgram theorem, there is a unique solution $(w, \vartheta) \in H^2_{\Gamma_0} \cap H^1_{\Gamma_1} \times H^1$ such that (3.16) is satisfied for all $(\widetilde{w}, \widetilde{\vartheta}) \in H^2_{\Gamma_0} \cap H^1_{\Gamma_1} \times H^1$. This implies that there exists a unique weak solution $\vartheta \in H^1$ of the following elliptic boundary-value problem

$$
\begin{cases}
c_\gamma \vartheta - c_\mu \Delta\vartheta = f_1(w, \widehat{w}, \widehat{\vartheta}, \widehat{\eta}) & \text{on } \Omega \\
\lambda_1 c_\mu \dfrac{\partial\vartheta}{\partial n} + \lambda_2 \vartheta = f_2(\widehat{\eta}) & \text{on } \Gamma
\end{cases}
$$

where

$$
\begin{aligned}
f_1(w, \widehat{w}, \widehat{\vartheta}, \widehat{\eta}) &= \alpha\Delta w + \beta\widehat{\vartheta} - \alpha\Delta\widehat{w} - \int_0^\infty \mu'(s) \int_0^s e^{\tau - s}\Delta\widehat{\eta}\, d\tau\, ds \\
&+ \int_0^\infty \gamma'(s) \int_0^s e^{\tau - s}\widehat{\eta}\, d\tau\, ds \in L^2
\end{aligned}
$$

and

$$
f_2(\widehat{\eta}) = \lambda_1 \left\{ \int_0^\infty \mu'(s) \int_0^s e^{\tau - s}\, \frac{\partial\widehat{\eta}^t(\tau)}{\partial n}\, d\tau \right\}.
$$

By the regularity theorem [**10**], it follows that $\vartheta \in H^2$, making sense to the boundary conditions. By (3.7), substituting v from $(3.12)_1$ into $(3.13)_1$, we have

$$\mathcal{B}\left(g_\infty w - \int_0^\infty g'(s)\left[(w-\widehat{w})(1-e^{-s}) + \int_0^s e^{\tau - s}\widehat{\psi}(\tau)\, d\tau\right] ds\right) + \alpha\vartheta = 0\,.$$

Moreover, $w \in H^2_{\Gamma_0} \cap H^1_{\Gamma_1}$ is a weak solution of the following elliptic boundary-value problem

$$\begin{cases} w + c_g \Delta^2 w = f_3(\vartheta, \widehat{v}, \widehat{w}, \widehat{\psi}) & \text{on } \Omega \\ w = \dfrac{\partial w}{\partial n} = 0 & \text{on } \Gamma_0 \\ w = 0\,, \qquad c_g \mathcal{B}(w) + \mathcal{B}(f_4(\widehat{w}, \widehat{\psi})) = -\alpha\vartheta \in H^{3/2}(\Gamma_1) & \text{on } \Gamma_1 \end{cases}$$

where

$$f_3(\vartheta, \widehat{v}, \widehat{w}, \widehat{\psi}) = -\alpha\Delta\vartheta + \widehat{v} + \widehat{w} + \int_0^\infty g'(s)\, \Delta^2\left[(e^{-s} - 1)\, \widehat{\psi} + \int_0^s e^{\tau - s}\widehat{\psi}\, d\tau\right] ds \in L^2.$$

and

$$f_4(\widehat{w}, \widehat{\psi}) = -\int_0^\infty g'(s)\left[(e^{-s} - 1)\, \widehat{w} + \int_0^s e^{\tau - s}\widehat{\psi}^t(\tau)\, d\tau\right] ds\,.$$

By the regularity theorem [**10**], we obtain $w \in H^4 \cap H^2_{\Gamma_0} \cap H^1_{\Gamma_1}$ and then $v = w - \widehat{w} \in H^2_{\Gamma_0} \cap H^1_{\Gamma_1}$. From $(3.12)_2$ and $(3.12)_3$ we obtain, respectively,

$$-\int_0^\infty g'(s)\, \Delta^2 \psi(s)\, ds\,, \qquad \int_0^\infty [\mu'(s)\, \Delta\eta(s) - \gamma'(s)\, \eta(s)]\, ds \in L^2.$$

By $(3.13)_1$ and $(3.12)_4$, we have $\psi \in M$. Analogously, $\eta \in N$. ■

Remark 3.3. If $\beta = 0$ and $g' \equiv \gamma' \equiv 0$ then well-posedness is proved also in the thermoelastic plate involving the Gurtin-Pipkin heat flow theory.

4 - Asymptotic behavior

In the sequel we take into consideration the asymptotic behavior of solutions $z(t) = T(t)\, z_0$ of (3.4). By means of Lemma 2.1, we prove the exponential stability of the semigroup generated by $\mathcal{A}$.

Theorem 4.1. *Suppose that* g, μ *and* γ *satisfy conditions from* (h1) *to* (h6). *Then* $T(t) = e^{t\mathcal{A}}$ *is exponentially stable.*

Proof. Let $Z_{\mathbb{C}}$ and $\mathcal{A}_{\mathbb{C}}$ be the complexification of Z and $\mathcal{A}$, respectively. First, we prove that $S(t)$, the contraction semigroup on $Z_{\mathbb{C}}$ generated by $\mathcal{A}_{\mathbb{C}}$, is exponentially stable. We use the contradiction argument assuming that the conclusion of Lemma 2.1 is not true. Thus, we consider the case when (2.4) fails to hold. Namely, there exists a sequence of $\beta_n \in \mathbb{R}$ and a sequence of $z_n = (w_n, v_n, \vartheta_n, \psi_n, \eta_n)^T \in D(\mathcal{A}_{\mathbb{C}})$, such that

$$\lim_{n\to+\infty} \|(i\beta_n I - \mathcal{A}_{\mathbb{C}})\, z_n\|_{Z_{\mathbb{C}}} = 0\,, \qquad \|z_n\|_{Z_{\mathbb{C}}} = 1 \ \ \forall n \in \mathbb{N}\,. \tag{4.1}$$

As $n \to \infty$, the limit in (4.1) is equivalent to

$$a_{\mathbb{C}}(i\beta_n w_n - v_n) \to 0 \quad \text{in } U_{\mathbb{C}} \tag{4.2}$$

$$i\beta_n v_n + g_\infty \Delta^2 w_n - \int_0^\infty g'(s)\, \Delta^2 \psi_n(s)\, ds + \alpha \Delta \vartheta_n \to 0 \quad \text{in } V_{\mathbb{C}} \tag{4.3}$$

$$i\beta_n \vartheta_n + \beta \vartheta_n + \int_0^\infty [\mu'(s)\, \Delta \eta_n - \gamma'(s)\, \eta_n]\, ds - \alpha \Delta v_n \to 0 \quad \text{in } \Theta_{\mathbb{C}} \tag{4.4}$$

$$i\beta_n \psi_n - v_n + \partial_s \psi_n \to 0 \quad \text{in } M_{\mathbb{C}} \tag{4.5}$$

$$i\beta_n \eta_n - \vartheta_n + \partial_s \eta_n \to 0 \quad \text{in } N_{\mathbb{C}} \tag{4.6}$$

where $\partial_s = \dfrac{\partial}{\partial s}$. Denoting by $\|\cdot\|_{\mathbb{C}}$ the norm of $L^2_{\mathbb{C}}$, the complexification of L^2, it follows that

$$\begin{aligned} &\mathrm{Re}\,\langle (i\beta_n I - \mathcal{A}_{\mathbb{C}})\, z_n, z_n\rangle_{Z_{\mathbb{C}}} = -\mathrm{Re}\,\langle \mathcal{A}_{\mathbb{C}} z_n, z_n\rangle_{\mathbb{C}} \\ &= \beta \|\vartheta_n\|^2_{\mathbb{C}} + \int_\Gamma \left[\int_0^\infty \mu'(s)\, \frac{\partial \eta_n(s)}{\partial n}\, ds \right] \vartheta_n\, d\Gamma + \frac{1}{2} \int_0^\infty g''(s)\, a_{\mathbb{C}}(\psi_n(s))\, ds \\ &\quad + \frac{1}{2} \int_0^\infty [\mu''(s) \|\nabla \eta_n(s)\|^2_{\mathbb{C}} + \gamma''(s) \|\eta_n(s)\|^2_{\mathbb{C}} +\, ds \underset{n\to\infty}{\longrightarrow} 0\,. \end{aligned} \tag{4.7}$$

Here, by virtue of (h3), each term is nonnegative and then tends to zero. Moreover, if $\lambda_1 = 0$ and $\lambda_2 \neq 0$, by virtue of (h4)-(h6) we obtain

$$\|\vartheta_n\|_{\Theta_{\mathbb{C}}} \underset{n\to\infty}{\longrightarrow} 0\,, \qquad \|\psi_n\|_{M_{\mathbb{C}}} \underset{n\to\infty}{\longrightarrow} 0\,, \qquad \|\eta_n\|_{N_{\mathbb{C}}} \underset{n\to\infty}{\longrightarrow} 0\,. \tag{4.8}$$

When $\lambda_1, \lambda_2 > 0$, we have in addition

$$\|\vartheta_n\|_{L^2_{\mathbb{C}}(\Gamma)} \xrightarrow[n\to\infty]{} 0 . \tag{4.9}$$

As a consequence of (4.8),

$$\|w_n\|^2_{U_{\mathbb{C}}} + \|v_n\|^2_{V_{\mathbb{C}}} \xrightarrow[n\to\infty]{} 1 . \tag{4.10}$$

On the other hand, from (4.2) and (2.2) we infer

$$i\beta_n w_n - v_n \xrightarrow[n\to\infty]{} 0 \qquad \text{in } V_{\mathbb{C}} = L^2_{\mathbb{C}},$$

so that

$$i\beta_n \langle w_n, v_n\rangle_{\mathbb{C}} - \|v_n\|^2_{V_{\mathbb{C}}} \xrightarrow[n\to\infty]{} 0 . \tag{4.11}$$

Starting from (4.3) we obtain

$$\begin{aligned}
&i\beta_n \langle v_n, w_n\rangle_{\mathbb{C}} + g_\infty \langle \Delta^2 w_n, w_n\rangle_{\mathbb{C}} \\
&- \int_0^\infty g'(s) \langle \Delta^2 \psi_n(s), w_n\rangle_{\mathbb{C}}\, ds + \alpha \langle \Delta\vartheta_n, w_n\rangle_{\mathbb{C}} \\
&= i\beta_n \langle v_n, w_n\rangle_{\mathbb{C}} + \|w_n\|^2_{U_{\mathbb{C}}} - \int_0^\infty g'(s)\, a_{\mathbb{C}}(\psi_n(s), w_n)\, ds + \alpha\langle \vartheta_n, \Delta w_n\rangle_{\mathbb{C}} \xrightarrow[n\to\infty]{} 0 .
\end{aligned}$$

The last term converges to zero because of (4.8) and $\|\Delta w_n\|_{\mathbb{C}} \leqslant 1$. Moreover, from (4.8) and (4.10) we have

$$\begin{aligned}
\left| \int_0^\infty g'(s)\, a_{\mathbb{C}}(\psi_n(s), w_n)\, ds \right| &\leqslant [a_{\mathbb{C}}(w_n)]^{1/2} \int_0^\infty -g'(s) [a_{\mathbb{C}}(\psi_n(s))]^{1/2}\, ds \\
&\leqslant \left(\frac{1-g_\infty}{g_\infty} \right)^{1/2} \|\psi_n\|_{M_{\mathbb{C}}} \xrightarrow[n\to\infty]{} 0 .
\end{aligned}$$

Thereby

$$i\beta_n \langle v_n, w_n\rangle_{\mathbb{C}} + \|w_n\|^2_{U_{\mathbb{C}}} \xrightarrow[n\to\infty]{} 0$$

and adding to it the complex conjugate of (4.11) we get

$$\|w_n\|^2_{U_{\mathbb{C}}} - \|v_n\|^2_{V_{\mathbb{C}}} \xrightarrow[n\to\infty]{} 0 . \tag{4.12}$$

By comparing this limit with (4.10) it follows that

$$\|w_n\|^2_{U_{\mathbb{C}}} \xrightarrow[n\to\infty]{} \frac{1}{2}, \qquad \|v_n\|^2_{V_{\mathbb{C}}} \xrightarrow[n\to\infty]{} \frac{1}{2} . \tag{4.13}$$

Now, we claim $|\beta_n| \geq \varepsilon > 0$, for all n. Otherwise, from (4.2) we infer that $v_n \xrightarrow[n\to\infty]{} 0$ in $U_{\mathbb{C}}$ (at least a subsequence) and so does in $V_{\mathbb{C}}$. This contradicts $(4.13)_2$. We complete the proof showing that (4.13) leads to a contradiction. Indeed, we rewrite (4.5) in the form

$$\psi_n - \frac{v_n}{i\beta_n} + \frac{\partial_s \psi_n}{i\beta_n} \xrightarrow[n\to\infty]{} 0 \quad \text{in } M_{\mathbb{C}}. \tag{4.14}$$

Under conditions imposed on $g(s)$, we can easily check that $\dfrac{sv_n}{i\beta_n} \in M_{\mathbb{C}}$. Thus, the limit (4.14) yields

$$\begin{aligned} &\int_0^\infty sg'(s)\, a_{\mathbb{C}}\left(\psi_n, \frac{v_n}{i\beta_n}\right) ds - a_{\mathbb{C}}\left(\frac{v_n}{i\beta_n}\right) \int_0^\infty sg'(s)\, ds \\ &\quad + \frac{1}{i\beta_n} \int_0^\infty sg'(s)\, a_{\mathbb{C}}\left(\partial_s \psi_n, \frac{v_n}{i\beta_n}\right) ds \xrightarrow[n\to\infty]{} 0 . \end{aligned} \tag{4.15}$$

By paralleling [**12**] previous arguments imply the first and the third term in (4.15) converge to zero. Thus the second term in (4.15) also converges to zero, namely

$$a_{\mathbb{C}}\left(\frac{v_n}{i\beta_n}\right) \xrightarrow[n\to\infty]{} 0$$

and, by (4.2), we obtain

$$a_{\mathbb{C}}(w_n) \xrightarrow[n\to\infty]{} 0 .$$

This contradicts (4.13). As a consequence, $S(t)$ is exponentially stable: there exists a constant $\omega > 0$ such that

$$\|S(t)\,\sigma_0\|_{Z_{\mathbb{C}}} \leq e^{-\omega t}\|\sigma_0\|_{Z_{\mathbb{C}}}, \qquad \forall \sigma_0 = (x_0 + iy_0) \in Z_{\mathbb{C}}.$$

In particular, if $\sigma_0 = z_0 + i0$ then

$$\|S(t)\,\sigma_0\|_{Z_C} = \|T(t)\,z_0\|_X \leqslant e^{-\omega t}\|\sigma_0\|_{Z_C} = e^{-\omega t}\|z_0\|_X,$$

so proving the exponential stability of T. ■

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Abstract

We study herein a two–dimensional evolution problem arising in the theory of linear thermoviscoelasticity with hereditary heat conduction. Linear semigroup theory is used to etablish the well–posedness and the exponential decay of solutions. In spite of the presence of a convolution term, the original problem is transformed into an autonomous system by suitable choice of variables. In order to achieve the exponential stability, we assume that mechanical and thermal memory kernels decay exponentially for large time and have a weak singularity at the origin.

* * *