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Fokker-Planck approximation of the linear Boltzmann equation with inelastic scattering: study of the distribution function for charged particles subjected to an external electric field (**)

1 - Introduction

In a recent paper [1], C. R. Garibotti and G. Spiga developed a formalism for introduction of inelastic collision processes in the Boltzmann equation. A two-species mixture was considered, where one of the particles is endowed with two internal energy levels, and a non-linear system for such a mixture was written down. Under suitable hypotheses, a linear transport equation was then derived for the study of the diffusion of light test particles (*TP*), like electrons, in a medium of field particles (*FP*) such that both a loss and a gain of a fixed amount of energy is possible when a *TP* interacts with a *FP*.

In this paper our aim is:

i. *to study the collision integral of such equation*

ii. *to point out the connections with transport of electrons in a semiconductor* [4], [5]

iii. *to construct, under suitable assumptions, a Fokker-Planck approximation of this equation and to solve it in the case of charged TP subjected to an external electric field.*

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The paper is organized as follows. Section 2 is devoted to the description of the physical situation we deal with. In Section 3 a study of the collision integral follows: collision invariants, trend to equilibrium and equilibrium distribution functions are investigated. Then (Section 4) we prove the mathematical equivalence between our problem and transport of electrons in a semiconductor.

In the case of TP which interact with FP only by means of elastic scattering, it is usual in the Physics of Weakly Ionized Gases (PWIG) to adopt a Fokker-Planck approximation of the collision integral, which leads to a solvable equation for the distribution function (see [2], [3]). Here we want to extend such an approach to inelastic interactions and find the distribution function which include these effects. First of all, by means of a truncated spherical harmonic expansion of the distribution function, the transport equation is broken into a system for the first two components (Section 5). Then, by adopting the same procedure which is usual for elastic interactions, we construct a Fokker-Planck approximation to the inelastic collision terms (Section 6).

The equations we find (Section 7) are solvable and explicit solutions are shown for both hard sphere and Maxwell interactions, together with a discussion on the possiblity that *runaway* occurs.

2 - Outline of the problem

Consider a space homogeneous medium constituted by *field particles* (FP) with mass M, endowed with only one excited internal energy level. We call $\Delta E > 0$ the difference of internal energy between the excited and the fundamental level.

Through this medium, we consider *test particles* (TP), endowed with mass $m \ll M$ and charge $\mathcal{Q}$, which diffuse under the effect of an external electric field $\mathbf{E}$.

The TP are supposed to interact with the medium according to the following inelastic mechanism

$$TP + FP_1 \rightleftharpoons TP + FP_2 \tag{1}$$

where FP_1 and FP_2 are FP, respectively, at fundamental and excited level. We assume that the number density n of TP is much lower than the number density $\mathcal{N}$ of FP, so that the medium can be modelled as a fixed background.

This medium is supposed to be in thermodynamical equilibrium at temperature T. If we call $\mathcal{N}_1$ and $\mathcal{N}_2$, respectively, the number densities of the FP at fundamental and excited energy levels, according to statistical mechanics

$$\frac{\mathcal{N}_2}{\mathcal{N}_1} = \exp\left(-\frac{\Delta E}{kT}\right) \tag{2}$$

where k is the Boltzmann constant.

Observe that one can write:

$$\mathcal{N}_1 = \frac{\mathcal{N}\mathcal{E}}{1+\mathcal{E}} \qquad \mathcal{N}_2 = \frac{\mathcal{N}}{1+\mathcal{E}} \tag{3}$$

where $\mathcal{E} = \exp(\Delta E/kT)$.

Since we consider $M \gg m$, the distribution function $\mathcal{F}_k(\mathbf{v})$ for FP_k can be approximated by $\mathcal{N}_k \delta(\mathbf{v})$, that is a Maxwellian with $M \to \infty$.

In this case the kinetic equation for the distribution function f of TP may be written as follows [1]:

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{x}} + \frac{Q\mathbf{E}}{m}\cdot\frac{\partial f}{\partial \mathbf{v}} = \mathcal{J}(\mathbf{v}) \tag{4}$$

with

$$\mathcal{J}(\mathbf{v}) = \frac{\mathcal{N}_1}{v}\int v_+^2 I(v_+, \boldsymbol{\Omega}\cdot\boldsymbol{\Omega}')\, f(v_+\,\boldsymbol{\Omega}')\, d\boldsymbol{\Omega}'$$

$$+ \frac{\mathcal{N}_2}{v}\int U(v^2-\eta)\, v^2 I(v, \boldsymbol{\Omega}\cdot\boldsymbol{\Omega}')\, f(v_-\,\boldsymbol{\Omega}')\, d\boldsymbol{\Omega}'$$

$$-f(\mathbf{v})\,\frac{1}{v}\int [\mathcal{N}_2 v_+^2 I(v_+, \boldsymbol{\Omega}\cdot\boldsymbol{\Omega}') + \mathcal{N}_1 U(v^2-\eta)\, v^2 I(v, \boldsymbol{\Omega}\cdot\boldsymbol{\Omega}')]\, d\boldsymbol{\Omega}'$$

where $\eta = 2\Delta E/m$, $v_\pm = \sqrt{v^2 \pm \eta}$ and $I(v, \boldsymbol{\Omega}\cdot\boldsymbol{\Omega}')$ is the cross section of the inelastic interaction between TP and FP_1 ($\boldsymbol{\Omega}$ is a unit vector and U is the step function). As physically plausible, we assume that

$$\lim_{v\to 0} v^2 I(v, \boldsymbol{\Omega}\cdot\boldsymbol{\Omega}') = 0\,.$$

It is important to stress that equation (4) is reconsidered here under slightly different assumptions than in [1].

3 - Study of the inelastic collision integral

Let us now study collisional invariatns and trend to equilibrium, according to the collision integral $\mathfrak{J}(\mathbf{v})$.

For any arbitrary function $\phi(\mathbf{v})$ it is not difficult to show that

$$\int \phi(\mathbf{v})\, \mathfrak{J}(\mathbf{v})\, \mathrm{d}\mathbf{v} \tag{5}$$

$$= \int \mathrm{d}\boldsymbol{\Omega} \int \mathrm{d}\boldsymbol{\Omega}' \int_0^\infty v[\phi(v\Omega) - \phi(v_+\, \boldsymbol{\Omega}')][\mathcal{N}_1 f(v_+\, \boldsymbol{\Omega}') - \mathcal{N}_2 f(v\boldsymbol{\Omega})]\, v_+^2\, I(v_+\,,\, \boldsymbol{\Omega}\cdot\boldsymbol{\Omega}')\, \mathrm{d}v\,.$$

From (5) we find that any $\phi(v\boldsymbol{\Omega}) = \Phi(v^2)$ such that $\Phi(v^2 + \eta) = \Phi(v^2)$ is a collisional invariant. In particular, the case $\Phi(v^2) = 1$ corresponds to number conservation for *TP*.

Equation (5) applied to $\phi(\mathbf{v}) = \log[f(\mathbf{v})/\exp(-\dfrac{mv^2}{2kT})]$ gives

$$\int \phi(\mathbf{v})\, \mathfrak{J}(\mathbf{v})\, \mathrm{d}\mathbf{v}$$

$$= \int \mathrm{d}\boldsymbol{\Omega} \int \mathrm{d}\boldsymbol{\Omega}' \int_0^\infty v\{\log f(v\boldsymbol{\Omega}) - \log[f(v_+\, \boldsymbol{\Omega}')\, \mathcal{E}]\}$$

$$[f(v_+\, \boldsymbol{\Omega}')\, \mathcal{E} - f(v\boldsymbol{\Omega})]\, \mathcal{N}_2\, v_+^2\, I(v_+\,,\, \boldsymbol{\Omega}\cdot\boldsymbol{\Omega}')\, \mathrm{d}v$$

which means that, in the space homogeneous case with $\mathbf{E} = \mathbf{0}$

$$\frac{\mathrm{d}}{\mathrm{d}t} \int f(\mathbf{v})\, \phi(\mathbf{v})\, \mathrm{d}\mathbf{v} \leqslant 0$$

that is we find a Boltzmann inequality or an H theorem (see [3]) for the present equation.

At equilibrium, for any v, $\boldsymbol{\Omega}$ and $\boldsymbol{\Omega}'$ it has to be

$$f(v\boldsymbol{\Omega}) = f(v_+\, \boldsymbol{\Omega}')\, \mathcal{E}$$

which is satisfied by any $f(v\boldsymbol{\Omega}) = \Gamma(v^2)\exp(-\dfrac{mv^2}{2kT})$ such that $\Gamma(v^2 + \eta) = \Gamma(v^2)$.

4 - Equivalence with electron transport in semiconductors

The results shown in Section 3 are surprisingly similar with those found by A. Majorana in the field of electron transport in semiconductors ([4]). Indeed, we shall prove that equation (4) is mathematically equivalent to the equation utilized in [4] and [5].

First of all let us rewrite, in the space homogeneous and forceless case, equation (4) as follows

$$\frac{\partial}{\partial t}\tilde{f}(\mathbf{w}) = \tilde{\mathfrak{J}}(\mathbf{w}) \tag{4'}$$

where $\mathbf{w} = \mathbf{v}/\sqrt{\eta}$, $\tilde{f}(\mathbf{w}) = f(\mathbf{v})$ and

$$\tilde{\mathfrak{J}}(\mathbf{w}) = \frac{\mathcal{N}_1}{w}\int w_+^2\, \tilde{I}(w_+,\, \mathbf{\Omega}\cdot\mathbf{\Omega}')\, \tilde{f}(w_+\, \mathbf{\Omega}')\, d\mathbf{\Omega}'$$

$$+\frac{\mathcal{N}_2}{w}\int U(w-1)\, w^2 \tilde{I}(w,\, \mathbf{\Omega}\cdot\mathbf{\Omega}')\, \tilde{f}(w_-\, \mathbf{\Omega}')\, d\mathbf{\Omega}'$$

$$-\tilde{f}(\mathbf{w})\frac{1}{w}\int [\mathcal{N}_2 w_+^2\, \tilde{I}(w_+,\, \mathbf{\Omega}\cdot\mathbf{\Omega}') + \mathcal{N}_1\, U(w-1)\, w^2 \tilde{I}(w,\, \mathbf{\Omega}\cdot\mathbf{\Omega}')]\, d\mathbf{\Omega}'$$

with $w_\pm = \sqrt{w^2 \pm 1}$, $\tilde{I}(w,\, \mathbf{\Omega}\cdot\mathbf{\Omega}') = \sqrt{\eta}\, I(v,\, \mathbf{\Omega}\cdot\mathbf{\Omega}')$.

Now, by taking into account that

$$U(w \pm 1)\, \delta(w' - w_\pm) = 2w_\pm\, \delta(w'^2 - w^2 \mp 1)$$

it is possible to express $\tilde{\mathfrak{J}}(\mathbf{w})$ in the equivalent form

$$\tilde{\mathfrak{J}}(\mathbf{w}) = \mathcal{E}\int \mathcal{K}(w,\, w',\, \mathbf{\Omega}\cdot\mathbf{\Omega}')\, \delta(w'^2 - w^2 - 1)\, \tilde{f}(\mathbf{w}')\, d\mathbf{w}'$$

$$+\int \mathcal{K}(w',\, w,\, \mathbf{\Omega}\cdot\mathbf{\Omega}')\, \delta(w^2 - w'^2 - 1)\, \tilde{f}(\mathbf{w}')\, d\mathbf{w}'$$

$$-\tilde{f}(\mathbf{w})[\int \mathcal{K}(w,\, w',\, \mathbf{\Omega}\cdot\mathbf{\Omega}')\, \delta(w'^2 - w^2 - 1)\, d\mathbf{w}'$$

$$+\mathcal{E}\int \mathcal{K}(w',\, w,\, \mathbf{\Omega}\cdot\mathbf{\Omega}')\, \delta(w'^2 - w^2 + 1)\, d\mathbf{w}']$$

where $\mathcal{K}(w,\, w',\, \mathbf{\Omega}\cdot\mathbf{\Omega}') = \dfrac{2\mathcal{N}_2 w'}{w}\tilde{I}(w',\, \mathbf{\Omega}\cdot\mathbf{\Omega}')$.

In [4] and [5] A. Majorana considers free electrons (effective mass m^*) interacting with monochromatic phonons (energy $\hbar\omega_L$, where $\hbar = h/2\pi$ and

$\omega_L = 2\pi\nu_L$, being h Planck's constant and ν_L phonon frequency) of a semiconductor lattice at temperature T. In [5] the distribution function φ of electrons depends on the dimensionless wave vector

$$\mathbf{c} = c\mathbf{\Omega} = \mathbf{k}\,(\frac{\hbar}{2m^*\omega_L})^{\frac{1}{2}}$$

where $\mathbf{k}$ is the electron wave vector. The kinetic equation for the present problem, as reported in [5], reads as follows:

$$\frac{\partial\varphi}{\partial t} = Q(\mathbf{c})$$

where Q turns out to have exactly the same form as last expression of $\tilde{\mathfrak{J}}$ we have shown. One can observe the following correspondences:

$$\tilde{f} \to \varphi \qquad \mathbf{w} \to \mathbf{c} \qquad \mathcal{E} \to \exp(\frac{\hbar\omega_L}{kT})$$

while $\mathcal{K}$ corresponds to a suitable continuous positive kernel of the electron-phonon interaction. Moreover, by recalling the expression of $\mathcal{E}$ as a function of temperature, it is apparent that $\Delta E \to \hbar\omega_L$.

5 - Spherical harmonics expansion

As usual in PWIG [2], if both the spatial gradients and the electric field are small, we may resort to a truncated spherical harmonic expansion of $f(v\mathbf{\Omega})$:

$$f(v\mathbf{\Omega}) = N(v) + \mathbf{\Omega}\cdot\mathbf{J}(v) \tag{6}$$

where $\quad N(v) = \dfrac{1}{4\pi}\displaystyle\int f(v\mathbf{\Omega})\,\mathrm{d}\mathbf{\Omega} \qquad \mathbf{J}(v) = \dfrac{3}{4\pi}\displaystyle\int \mathbf{\Omega} f(v\mathbf{\Omega})\,\mathrm{d}\mathbf{\Omega}\,.$

By inserting (6) into (4) and projecting over 1 and $\mathbf{\Omega}$ we find the following system for $N(v)$ and $\mathbf{J}(v)$:

$$\begin{aligned}\frac{\partial N}{\partial t} + \frac{v}{3}\nabla\cdot\mathbf{J} + \frac{\mathcal{Q}}{m}\mathbf{E}\cdot\frac{1}{3v^2}\frac{\partial}{\partial v}(v^2\mathbf{J}) &= \frac{\mathcal{N}_1 v_+^2}{v} N(v_+)\, I_0(v_+) \\ + \mathcal{N}_2 v U(v^2-\eta)\, N(v_-)\, I_0(v) - N(v)\,\frac{1}{v}&[\mathcal{N}_2 v_+^2\, I_0(v_+) + \mathcal{N}_1\, U(v^2-\eta)\, v^2 I_0(v)]\end{aligned} \tag{7}$$

$$\frac{\partial \mathbf{J}}{\partial t} + v\nabla N + \frac{\mathcal{Q}}{m}\mathbf{E}\frac{\partial N}{\partial v} = \frac{\mathcal{N}_1 v_+^2}{v}\mathbf{J}(v_+)\, I_1(v_+)$$

(8)

$$+\mathcal{N}_2 v U(v^2-\eta)\,\mathbf{J}(v_-)\, I_1(v) - \mathbf{J}(v)\frac{1}{v}[\mathcal{N}_2 v_+^2 I_0(v_+) + \mathcal{N}_1 U(v^2-\eta)\, v^2 I_0(v)]$$

where $$I_l(v) = 2\pi\int_{-1}^{+1} I(v,\mu)\,\mu^l\,\mathrm{d}\mu \qquad l=0,1\,.$$

In view of the manipulations we are going to perform on the right hand sides of these equations, it is convenient to introduce

$$F(\xi) = N(v) \qquad \mathbf{G}(\xi) = \mathbf{J}(v) \qquad \sigma_k(\xi) = I_k(v)$$

where $\xi = v^2$, and rewrite equations (7) and (8) as follows:

(9)

$$\frac{\partial F}{\partial t} + \frac{\sqrt{\xi}}{3}\nabla\cdot\mathbf{G} + \frac{\mathcal{Q}}{m}\mathbf{E}\cdot\frac{2}{3\sqrt{\xi}}\frac{\partial}{\partial\xi}(\xi\mathbf{G}) = \left(\frac{\partial F}{\partial t}\right)_{\text{coll}} = \frac{\mathcal{N}_1}{\sqrt{\xi}}(\xi+\eta)\,F(\xi+\eta)\,\sigma_0(\xi+\eta)$$

$$+\mathcal{N}_2\sqrt{\xi}U(\xi-\eta)\,F(\xi-\eta)\,\sigma_0(\xi) - F(\xi)\frac{1}{\sqrt{\xi}}[\mathcal{N}_2(\xi+\eta)\,\sigma_0(\xi+\eta) + \mathcal{N}_1 U(\xi-\eta)\,\xi\sigma_0(\xi)]$$

(10)

$$\frac{\partial \mathbf{G}}{\partial t} + \sqrt{\xi}\nabla F + \frac{2\mathcal{Q}}{m}\mathbf{E}\sqrt{\xi}\frac{\partial F}{\partial\xi} = \left(\frac{\partial \mathbf{G}}{\partial t}\right)_{\text{coll}} = \frac{\mathcal{N}_1}{\sqrt{\xi}}(\xi+\eta)\,\mathbf{G}(\xi+\eta)\,\sigma_1(\xi+\eta)$$

$$+\mathcal{N}_2\sqrt{\xi}U(\xi-\eta)\,\mathbf{G}(\xi-\eta)\,\sigma_1(\xi) - \mathbf{G}(\xi)\frac{1}{\sqrt{\xi}}[\mathcal{N}_2(\xi+\eta)\,\sigma_0(\xi+\eta) + \mathcal{N}_1 U(\xi-\eta)\,\xi\sigma_0(\xi)]\,.$$

6 - The "Fokker-Plank" approximation

In the stationary and space homogeneous case equations (9) and (10) are rewritten as

(11)
$$\frac{\mathcal{Q}|\mathbf{E}|}{m}\frac{2}{3\sqrt{\xi}}\frac{\mathrm{d}}{\mathrm{d}\xi}(\xi G) = \left(\frac{\partial F}{\partial t}\right)_{\text{coll}}$$

(12)
$$\frac{\mathcal{Q}|\mathbf{E}|}{m}2\sqrt{\xi}\frac{\mathrm{d}F}{\mathrm{d}\xi} = \left(\frac{\partial G}{\partial t}\right)_{\text{coll}}$$

where $$G(\xi) = \mathbf{G}(\xi) \frac{\mathbf{E}}{|\mathbf{E}|} \qquad (\frac{\partial G}{\partial t})_{\text{coll}} = (\frac{\partial \mathbf{G}}{\partial t})_{\text{coll}} \cdot \frac{\mathbf{E}}{|\mathbf{E}|}$$

If η is much lower than the thermal mean square speed of a TP, that is $\eta \ll \frac{3kT}{m}$, it is possible to adopt a procedure (see [2]) which leads to a Fokker-Plank approximation for $(\partial F/\partial t)_{\text{coll}}$.

Consider an arbitrary smooth function $\Phi(\xi)$, one can easily show that

$$\int_0^\infty \sqrt{\xi}\,(\frac{\partial F}{\partial t})_{\text{coll}}\, \Phi(\xi)\, d\xi$$

$$= \int_0^\infty d\xi[\Phi(\xi) - \Phi(\xi+\eta)][\mathcal{N}_1 F(\xi+\eta) - \mathcal{N}_2 F(\xi)](\xi+\eta)\, \sigma_0(\xi+\eta)\, d\xi\,.$$

Now we expand the integrand of the right hand side in power series of η and retain only the terms up to η^2:

$$\int_0^\infty \sqrt{\xi}\,(\frac{\partial F}{\partial t})_{\text{coll}}\, \Phi(\xi)\, d\xi = -\frac{1}{2}\eta^2 \mathcal{N} \int_0^\infty \Phi'(\xi)[F(\xi)\frac{m}{2kT} + F'(\xi)]\, \xi\sigma_0(\xi)\, d\xi$$

and, integrating by parts

$$\int_0^\infty \sqrt{\xi}\,(\frac{\partial F}{\partial t})_{\text{coll}}\, \Phi(\xi)\, d\xi = \frac{1}{2}\eta^2 \mathcal{N} \int_0^\infty \Phi(\xi)\frac{d}{d\xi}\{[F(\xi)\frac{m}{2kT} + F'(\xi)]\, \xi\sigma_0(\xi)\}\, d\xi\,.$$

Since $\Phi(\xi)$ is arbitrary, we obtain the following Fokker-Planck expression of the collision term

$$(\frac{\partial F}{\partial t})_{\text{coll}} = \frac{1}{2}\eta^2 \mathcal{N} \frac{1}{\sqrt{\xi}}\frac{d}{d\xi}\{(F(\xi)\frac{m}{2kT} + F'(\xi)]\, \xi\sigma_0(\xi)\}\,. \tag{13}$$

We remark that $(\frac{\partial F}{\partial t})_{\text{coll}} = 0 \Leftrightarrow F = C\exp(-\frac{m}{2kT}\xi)$, that is the equilibrium distribution function for the present approximation is a Maxwellian (C is the normalization constant).

Observe that (11), together with (13), shows that $G = O(\eta^2)$ so that $(\partial G/\partial t)_{\text{coll}}$

is coherently approximated by

$$-\mathcal{N}\sqrt{\xi}\,\sigma_*(\xi)\,G(\xi) \tag{14}$$

where $\sigma_*(\xi) = \sigma_0(\xi) - \sigma_1(\xi)$.

7 - Solution to equations (11) and (12)

Equations (11) and (12) give

$$\frac{2}{3}\tilde{E}G(\xi) = \frac{1}{2}\eta^2\sigma_0(\xi)[F'(\xi) + \frac{m}{2kT}F(\xi)] \qquad 2\tilde{E}F'(\xi) = -\sigma_*(\xi)\,G(\xi)$$

where $\tilde{E} = \dfrac{\mathcal{Q}|\mathbf{E}|}{m\mathcal{N}}$.

By eliminating G the following equation for F is obtained:

$$[\frac{8}{3}\left(\frac{\tilde{E}}{\eta}\right)^2 \frac{1}{\sigma_*(\xi)\,\sigma_0(\xi)} + 1]\,F'(\xi) = -\frac{m}{2kT}F(\xi)$$

which gives $$F(\xi) = A\exp[-\frac{m}{2kT}\int_0^{\xi}(1 + \frac{8}{3}\left(\frac{\tilde{E}}{\eta}\right)^2 \frac{1}{\sigma_*(\xi')\,\sigma_0(\xi')})^{-1}\mathrm{d}\xi']$$

where A is given by the normalization condition

$$2\pi\int_0^{\infty}\sqrt{\xi}\,F(\xi)\,\mathrm{d}\xi = n\,.$$

Under certain assumptions on the cross sections, we shall show the explicit expression of such distribution function.

Suppose that the cross section is separable in energy and angle, that is

$$\sigma(\xi, \mu) = P(\xi)\,Q(\mu)\,.$$

The distribution function takes the form

$$F(\xi) = A\exp[-\frac{m}{2kT}\int_0^{\xi}(1 + \frac{8}{3}\left[\frac{\tilde{E}}{\eta P(\xi')}\right]^2 \frac{1}{c_* c_0})^{-1}\mathrm{d}\xi']$$

where $$c_0 = 2\pi\int_{-1}^{+1} Q(\mu)\,\mathrm{d}\mu \qquad c_* = 2\pi\int_{-1}^{+1} Q(\mu)(1-\mu)\,\mathrm{d}\mu\,.$$

It is well known (see [6] and references therein) that, in the stationary and homogeneous case, the transport equation for charged particles subjected to an external electric field has not, in general, a solution which can be normalized. The nonexistence of such a solution is called *runaway phenomenon.* The occurrence of runaway depends on how fast is the decay of the cross section, as a function of energy.

Here we shall consider two physically meaningful interaction laws which give rise to different situations, with respect to the possiblity that runaway occurs.

In the case of *hard sphere interactions*, that is $P(\xi) = 1$, we have:

$$F(\xi) = A \exp\left(-\frac{m\xi}{2\theta kT}\right) \qquad \text{where } \theta = 1 + \frac{8}{3}\left(\frac{\widetilde{E}}{\eta}\right)^2 \frac{1}{c_0 c_*}.$$

It is always possible to normalize this $F(\xi)$, no matter how large is the electric field (within the range of applicability of the spherical harmonics expansion, of course).

In this case of *Maxwell interactions*, that is $P(\xi) = 1/\sqrt{\xi}$, we have

$$F(\xi) = \left[1 + \frac{8}{3}\left(\frac{\widetilde{E}}{\eta}\right)^2 \frac{\xi}{c_0 c_*}\right]^{-\gamma} \qquad \text{where } \gamma = c_0 c_* \frac{3}{16}\frac{m}{kT}\left(\frac{\eta}{\widetilde{E}}\right)^2.$$

This $F(\xi)$ can be normalized only when the electric field is sufficiently small: $\widetilde{E} < \eta\sqrt{\frac{m}{kT}\frac{c_0 C_*}{8}}$, otherwise runaway phenomenon occurs.

In conclusion, we would like to point out that interesting extensions of the present approach could be the introduction of elastic scattering and/or a more realistic treatment of inelastic interaction, by keeping M/m finite. Such improvements of the model would allow comparisons with the well known results by Margenau and Druyvesteyn (see [2] and [3]), valid when only elastic interactions occur.

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Sommario

L'equazione lineare di Boltzmann per scattering inelastico è applicata allo studio della funzione di distribuzione per particelle cariche, soggette ad un campo elettrico esterno, in un mezzo ospite. Si studia l'integrale collisionale inelastico e si mettono in luce le connessioni col trasporto di elettroni nei semiconduttori. Viene poi sviluppata una approssimazione alla Fokker-Planck dell'equazione cinetica, che porta, nel caso spazialmente omogeneo, ad un sistema risolubile per i primi due momenti della funzione di distribuzione. Si forniscono e si commentano risultati espliciti per interazioni di interesse fisico.

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