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Some remarks on almost Hermitian manifolds (**)

We begin with two theorems.

Theorem 1 (Blair [1]). *If M is a 4-dimensional almost Kähler manifold of constant curvature, then M is a Kähler manifold.*

Theorem 2 (Olszak [4]). *If M is an almost Kähler manifold of constant curvature and dimension* $2n \geq 8$, *then M is a Kähler manifold.*

In [3] J. J. Konderak gave an example of a 4-dimensional almost Hermitian flat manifold which is not Hermitian. Consequently this manifold is not Kähler and we can make

Remark 1. *The theorem due to Blair is no more true, if M is assumed to be a 4-dimensional almost Hermitian manifold of constant curvature.*

Now we generalize Konderak's example in higher dimensions.

Since R^{2n-4} $(n \geq 3)$ can be identified with C^{n-2}, then R^{2n-4} can be regarded as an almost Hermitian manifold. Now let R_K^4 be the manifold introduced by J. J. Konderak in [3]. Consider the product manifold $M_K = R^{2n-4} \times R_K^4$.

Standard procedures allow us to define an almost Hermitian structure on M_K. It is easy to check that M_K is a $2n$-dimensional flat almost Hermitian manifold, which is not Hermitian.

Thus we are able to make

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Remark 2. *Theorem 2 due to Olszak is no more true, if M is assumed to be an almost Hermitian manifold of constant curvature and dimension* $2n \geqslant 8$.

An almost Hermitian manifold M with almost Hermitian structure (J, g) is called an *almost L-manifold* (Friedland and Hsiung [2]) if we have

$$(\nabla_X \nabla_Y - \nabla_Y \nabla_X) J = 0$$

for all vector fields X and Y on M, where ∇ is the Levi-Civita connection of g. Clearly, all Kähler manifolds are almost L-manifolds.

It is immediate to see that the manifold R_K^4 introduced by Konderak and also $M_k = R^{2n-4} \times R_k^4$ are almost L-manifolds. So we are able to prove

Proposition 1. *There exist almost L-manifolds which are not Kähler.*

References

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Sommario

Si prova che due teoremi noti per le varietà quasi Kähleriane non possono essere estesi alle varietà quasi Hermitiane.

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