Riv. Mat. Univ. Parma (5) 3 (1994), 123-142

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Submanifolds of almost hermitian manifolds (**)

1 - Introduction

The theory of Kähler manifolds is by now very rich in interesting results and continues to be one of the most important areas of differential geometry. In the last forty years a special attention was devoted to the study of submanifolds of Kähler manifolds. We have, presently, a large number of very beautiful theorems in the case when the ambient space is a Kähler manifold of constant holomorphic sectional curvature, i.e. complex euclidean spaces, complex projective spaces, complex hyperbolic spaces. Good reference books on this theme are these by K. Yano - M. Kon, B. Y. Chen, A. Bejancu, M. Dajczer etc. There are also several survey articles, see for examples papers of K. Ogiue [78], [79], A. Gray [51] etc.

On other hand there exist compact (almost) complex manifolds which do not bear any (almost) Kähler metric. The complex Hopf manifold is but one example. Their geometry is less understood than the Kähler's one and still represent a challenge. Hence, it is not surprising that the study of submanifolds in non-Kähler ambient space was a point of interest for many geometers in the last 30 years. The first notable results of this theory were obtained in the sixties by A. Gray and by L. Vanhecke in the seventies. In the last decade many researches have been done for the theory of submanifolds of locally conformal Kähler manifolds (see references).

In this expository paper we will refer to submanifolds of almost hermitian, not necessarily Kähler manifolds. We will present a few interesting results for

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(**) Received November 15, 1994. AMS classification 53 C 40.

this subject and will try to give a fairly good amount of references for this subject, although we know that our list can be completed.

As it is well-known, the study of a submanifold M of an almost hermitian manifold $\widetilde{M}$ is essentially dictated by the behaviour the tangent bundle TM with respect to the actions of the almost complex structure J of $\widetilde{M}$. Consequently, many authors studied intensively the following three cases: 1) holomorphic submanifolds (invariant with respect to J); 2) antiholomorphic submanifolds; 3) CR-submanifolds.

Very recently, B. Y. Chen introduced the notion of slant submanifold (see survey paper [**18**])

Accordingly, after reviewing in Section **2**, some preliminaries of hermitian geometry and theory of riemannian submanifolds, Section **3**, **4**, **5** will be devoted to the three above mentionned classes of submanifolds.

2 - Some definitions and formulas

We say that (M, J, g) is an *almost hermitian manifold* (AH-manifold) if the tangent bundle of M has an almost complex structure J (i.e. $J^2 = -1$) and a riemannian metric g such that $g(JX, JY) = g(X, Y)$ for all $X, Y \in \Gamma(TM)$. Here $\Gamma(TM)$ denotes the Lie algebra of vector fields on M. The manifold M is orientable and even-dimensional $2m$.

Now we recall some important classes of AH-manifolds.

An AH-manifold with J integrable is called a *Hermitian manifold* (H-manifold).

The book of Kobayashi and Nomizu [**69**], is a good reference for definitions and fundamental properties of AH-manifolds. The fundamental 2-form Ω a AH-manifold is defined by $\Omega(X, Y) = g(X, JY)$, for any $X, Y \in \Gamma(TM)$.

An *AH*-manifold is called an *almost Kähler manifold* (AK-manifold) if Ω is closed.

Using the Levi-Civita connection ∇ of an AH-manifold (M, J, g), we define a *nearly Kähler manifold* (NK-manifold) and a *quasi-Kähler manifold* (QK-manifold) by the conditions:

$$\text{(NK)}: \ (\nabla_X J)Y + (\nabla_Y J)X = 0$$

$$\text{(QK)}: \ (\nabla_X J)Y + (\nabla_{JX} Y)JY = 0$$

for all $X, Y \in \Gamma(TM)$.

An AH-manifold is Kähler iff $\nabla J = 0$. An AH-manifold is hermitian iff:

$$(H) \qquad (\nabla_X J) Y - (\nabla_{JX} J) JY = 0 .$$

We have the strict inclusion relations

$$K \subset AK \subset QK \subset AH \qquad K \subset NK \subset QK \qquad K \subset H \subset AH$$

(see [42], [43], [110]).

For a lot local formulas in the differential complex geometry, see the book of K. Yano [116].

The Kähler manifolds form a very interesting class of manifolds in differential geometry. It is well known that the geometrical and topological structure of these manifolds is very rich.

The nearly Kähler manifolds, which are not Kähler, are perhaps the most interesting nonintegrable almost hermitian manifolds (see [47], [50]). For the different relations between all the given classes of AH-manifolds and examples, see the papers of Gray [42], [43], [51], Abbena [2], Cordero-Fernandez-de Leon [22], Gray-Vanhecke [52] and Vanhecke [110].

Now we define the class of *locally conformal Kähler manifolds* (l.c.K-manifolds), which has been developped mainly in the last 20 years.

I. Vaisman gave the main properties of l.c.K-manifolds in a series of papers (see [106]-[108]).

A H-manifolds is called l.c.K-manifold (M, J, g) if each of its points has a neighbourhood U on which the restriction of the metric g is conformal with a Kähler metric g' on U, such that $g|_U = \exp(f_U) g'$ where f is a differentiable function on U.

Equivalently, a H-manifold is l.c.K-manifold iff there exists on M a closed 1-form ω, called the Lee form, such that $d\Omega = \omega \wedge \Omega$.

A l.c.K-manifold with parallel Lee form is called a *generalized Hopf manifold* (g.H-manifold). We shall suppose $\omega \neq 0$ everywhere. For a g.H-manifold the Lee 1-form ω has constant length: let $|\omega| = \frac{c}{2}$. We denote $u = \frac{\omega}{2}$. Let U be the vector field defined by $g(U, X) = u(X)$ for all X tangent to M and $V = JU$. For a survey in theory of l.c.K-manifolds, see L. Ornea [83].

A rigorous study of the curvature of the AH-manifolds was given by A. Gray and L. M. Hervella [53], F. Tricerri and L. Vanhecke [103], [104]. For many recent valuable results in the geometry of AH-manifolds see M. Falcitelli, A. Farinola and S. Salamon [38], V. F. Kirichenko [68]. We say that an AH-manifold M is a *para-Kähler manifold* (cf. G. B. Rizza [90]) or a *F-space* (cf. [96], [97]) if the curvature tensor R satisfies

$$R(X, Y, Z, W) = R(X, Y, JZ, JW) \tag{2.1}$$

for all $X, Y, Z, W \in \Gamma(TM)$. There exists examples of flat para-Kähler manifolds which are not Kähler manifolds (see [103], [70]).

L. Vanhecke considers in [109] the class of AH-manifolds with J-invariant riemannian curvature tensor, so that R satisfies

$$R(X, Y, Z, W) = R(JX, JY, JZ, JW) \tag{2.2}$$

for all $X, Y, Z, W \in \Gamma(TM)$ and he called them RK-manifolds.

Any nearly Kähler manifold and any para-Kähler manifold is an RK-manifold.

Now, we say that an AH-manifold (M, J, g) has *pointwise constant type* if, for any $p \in M$ and any $X, Y, Z \in T_pM$ such that

$$g(X, Y) = g(X, Z) = g(X, JY) = g(X, JZ) = O$$

and $g(X, Y) = g(Z, Z) = 1$, we have $\lambda(X, Y) = \lambda(X, Z)$, where

$$\lambda(X, Y) = R(X, Y, X, Y) - R(X, Y, JX, JY). \tag{2.3}$$

The notion of constant type has been introduced by A. Gray for NK-manifolds ([47]).

If M is an RK-manifold, then M has pointwise constant type if there exists a differentiable funcion α on M (cf. [109], [110]) such that

$$\lambda(X, Y) = \alpha\{g(X, X)g(Y, Y) - g(X, Y)^2 - g(X, JY)^2\} \tag{2.4}$$

for all $X, Y \in \Gamma(TM)$. Moreover, M has *global constant type* if and only if α is constant on M. The function α is said to be the constant type of M.

Then, L. Vanhecke defines a *generalized complex space form* to be an RK-manifold of constant holomorphic sectional curvature μ and constant type α. The generalized complex space forms form a proper extension of the class of complex space forms because there are a lot of examples of generalized complex spaces which are not Kähler spaces (see [110], [103], [68], [80], [104]). Let $M(\mu, \alpha)$ be a generalized complex space form of constant holomorphic sectional curvature μ

and constant type α. Then the curvature tensor R of $M(\mu, \alpha)$ satisfies

$$(2.5)\qquad \begin{aligned} R(X, Y, Z, W) = &\frac{1}{4}(\mu + 3\alpha)\{g(X, Z)g(Y, W) - g(X, W)g(Y, Z)\} \\ &+ \frac{1}{4}(\mu - \alpha)\{g(X, JZ)g(Y, JW) - g(X, JW)g(Y, JZ)\} \\ &+ \frac{1}{2}(\mu - \alpha)g(X, JY)g(Z, JW)\}. \end{aligned}$$

We note that if $\mu = \alpha$ then $M(\mu, \alpha)$ is a space of constant curvature.

3 - Holomorphic submanifolds of a AH-manifold

Let $(\widetilde{M}, g)$ be a m-dimensional Riemann manifold and let M be a n-dimensional manifold immersed in $\widetilde{M}$. Since we are dealing with a local study, we may assume that M is imbedded in $\widetilde{M}$. If $F\colon M \to \widetilde{M}$ is an immersion of M in $\widetilde{M}$, for each $p \in M$ we identify the tangent space T_pM with $dF(T_pM) \subset T_{F(p)}\widetilde{M}$ by means of dF and M is called a *submanifold* of $\widetilde{M}$. Throughout the paper we consider that M is isometrically immersed and we continue to denote by g the riemannian metric induced on M.

Let $(\widetilde{M}, J, g)$ be an AH-manifold. A submanifold M in $\widetilde{M}$ is called a *holomorphic submanifold* of $\widetilde{M}$ if T_pM is invariant by J, i.e. if we have $J(T_pM) = T_pM$, for each $p \in M$. The almost complex structure induced on M is still denoted by J.

The study of holomorphic submanifold of a Kähler manifold was initiated by E. Calabi in the early of 1950 (cf. [12]). For survey papers on this subject, see K. Ogiue [78], [79]. In this article we consider only holomorphic submanifolds of an AH-manifold without Kähler condition.

Now, we denote by $TM^{\perp}$ the normal bundle of M and by $\Gamma(TM^{\perp})$ the module of sections of the vector bundle $TM^{\perp}$. Of course, if M is a holomorphic submanifold of an AH-manifold, $T_pM^{\perp}$ is invariant by J for each $p \in M$.

We denote by $\widetilde{\nabla}$ and ∇ the Levi-Civita connections on $\widetilde{M}$ and M respectively. Then we have the Gauss formula

$$(3.1)\qquad \widetilde{\nabla}_X Y = \nabla_X Y + h(X, Y) \qquad X, Y \in \Gamma(TM)$$

and the Weingarten formula

$$(3.2)\qquad \widetilde{\nabla}_X \xi = -A_\xi X + \nabla^{\perp}_X \xi \qquad X \in \Gamma(TM),\ \ \xi \in \Gamma(TM^{\perp}).$$

Here A_ξ is the Weingarten's operator with respect to ξ, $\nabla^{\perp}$ denote the connection induced on the normal bundle $TM^{\perp}$ and h is *the second fundamental* *form* of the submanifold M in $\widetilde{M}$. We define the *mean curvature vector* H of M by $H = \frac{1}{n}$. Trace h, which is a normal vector field to M. If $H = 0$ on M we say that M is a *minimal submanifold* of $\widetilde{M}$. The submanifold M is said to be *totally geodesic* in $\widetilde{M}$ if its second fundamental form vanishes identically, i.e. $h = 0$ or equivalently $A_\xi = 0$, for any $\xi \in \Gamma(TM^\perp)$ (cf. [69], p. 25).

A submanifold M is said to be *totally umbilical* in $\widetilde{M}$ if we have

$$h(X, Y) = g(X, Y)H \qquad \text{for any } X, Y \in \Gamma(TM). \tag{3.3}$$

Finally, M is called a submanifold with *parallel second fundamental form*, if we have $\overline{\nabla} h = 0$.

The first interesting results on holomorphic submanifolds of an AH-manifold without Kähler condition were obtained by A. Gray.

Theorem 1 [42]. *If $\widetilde{M}$ is almost Kähler, nearly Kähler, quasi-Kähler or hermitian, then any holomorphic submanifold M of $\widetilde{M}$ has the same property. Moreover, any holomorphic submanifold M of a quasi-Kähler (in particular almost Kähler or nearly Kähler) manifold $\widetilde{M}$ is a minimal submanifold.*

Theorem 2 [44]. *Let $\widetilde{M}$ be a complete simply connected quasi-Kähler manifold with non-positive sectional curvature. If M is a closed isometrically immersed holomorphic submanifold of real dimension $2k$, then M has the homotopy type of a CW-complex with no cells of dimension greater than k.*

We observe that in general a holomorphic submanifold of a H-manifold is not minimal. However, there are examples of H-manifolds such that their holomorphic submanifolds are totally geodesic.

E. Calabi and B. Eckmann (cf. [14]) proved that the product of two odd-dimensional spheres carriers a non-Kähler hermitian structure, (Calabi-Eckmann manifold). Moreover, A. Morimoto (cf. [74]) proved that the product of two sasakian manifolds is a non-Kähler hermitian manifold.

For other interesting generalizations of Calabi-Eckmann manifolds see the papers of Blair, Ludden and Yano [10] and of K. Abe [1].

J. M. Sierra (cf. [100]) proved that if $\widetilde{M}$ is a product of two sasakian manifolds (in particular a Calabi-Eckmann manifold), any holomorphic submanifold of $\widetilde{M}$ is totally geodesic and necessarily minimal (cf. [65]).

Now we return to the holomorphic submanifolds of an AH-manifold and suppose that the second fundamental from h is *complex bilinear*, i.e. h satisfies

$$h(JX, Y) = h(X, JY) = Jh(X, Y) \qquad X, Y \in \Gamma(TM). \tag{3.4}$$

This condition is equivalent with

$$(3.5) \qquad A_{J\xi}X = JA_{\xi}X \qquad A_{\xi}JX = -JA_{\xi}X$$

for all $X \in \Gamma(TM)$ and $\xi \in \Gamma(TM^{\perp})$.

The holomorphic submanifolds of Kähler or nearly Kähler manifolds are such that the second fundamental form is complex bilinear (see, for example, [71] and [112]. It is easy to see that, if the second fundamental form of a holomorphic submanifold M of an AH-manifold $\tilde{M}$ is complex bilinear, then M is minimal in $\tilde{M}$.

Recently some authors consider a more general concept. If M is an AH-manifold and $\tilde{M}$ is a Riemannian manifold, a map $\varphi: M \to \tilde{M}$ is called *pluriharmonic* or (1, 1)-*geodesic* if the second fundamental form of φ satisfies

$$(3.6) \qquad h(X, JY) = h(JX, Y) \qquad \text{for all } X, Y \in \Gamma(TM).$$

For references on the pluriharmonic maps a good survey is the paper by J. Eells and L. Lemaire [34]. See also the book by M. Dajczer [23]. In recent papers [32], [60], K. L. Duggal, A. M. Pastore and the author study this concept in the metric contact geometry.

Now we give some theorems for holomorphic submanifolds, such that the second fundamental form is complex bilinear.

We recall that a submanifold M of a AH-manifold $\tilde{M}$ is called an *invariant submanifold* if each tangent space is invariant under the curvature transformation.

Theorem 3 [96], [97]. *Let M be a holomorphic hypersurface of a locally symmetric NK-manifold $\tilde{M}$ such that the second fundamental form is complex bilinear. If M is Ricci parallel and curvature invariant, then M is locally symmetric.*

Theorem 4 [113]. *Let $\tilde{M}(\mu, \alpha)$ be a generalized complex space form and M a holomorphic submanifold of $\tilde{M}$ satisfying* (3.4). *Then the following conditions are equivalent:*

i. *the normal connection of M is trivial*

ii. *$\mu = \alpha$ and M is totally geodesic in $\tilde{M}$ if the real codimension is* > 2.

These theorems generalize some well-known theorems by K. Nomizu and B. Smyth [77], [101] for complex hypersurfaces of a Kähler manifold with constant holomorpic curvature.

Theorem 5 [**114**]. *Let M be a holomorphic submanifold of a para-Kähler manifold $\widetilde{M}$. If the second fundamental form of M in $\widetilde{M}$ is complex bilinear, then M is a para-Kähler manifold. Moreover, the normal connection is flat if and only if the Ricci tensors S and $\widetilde{S}$ of M and $\widetilde{M}$ satisfy the relation $S(X, Y) = \widetilde{S}(X, Y)$ for all $X, Y \in \Gamma(TM)$.*

Corollary 1 [**114**]. *Let M be a holomorphic submanifold of a para-Kähler Einstein manifold $\widetilde{M}$. If the second fundamental form of M in $\widetilde{M}$ is complex bilinear, then M is also an Einstein manifold. Moreover, M and $\widetilde{M}$ have the same scalar curvature.*

F. Tricerri and L. Vanhecke defined in [**103**], [**104**] a Bochner tensor $\widetilde{B}$ for an AH-manifold $\widetilde{M}$ and proved some interesting results on the holomorphic submanifolds in $\widetilde{M}$ if $\widetilde{B}$ vanishes.

Concerning the geometry of six-dimensional NK-manifolds, we can give some nice results. Of course, S^6 is a standard example of generalized complex space form which is not a complex space. The curvature tensor of a six-dimensional NK-manifold is pointwise a linear combination of the curvature tensor of S^6 and the curvature of a Calabi-Yau space (cf. [38]). A. Gray [**45**] proved that S^6 with standard nearly Kähler structure has no four-dimensional almost complex submanifolds.

Concerning the 2-dimensional holomorphic submanifolds of a 6-dimensional sphere we have:

Theorem 6 [**98**], [**99**]. *Let M be a 2-dimensional holomorphic submanifold of the 6-dimensional sphere S^6 with usual nearly Kähler structure. If the Gauss curvature K of M is constant on M, then $k = 1$, or $\frac{1}{6}$ or 0.*

K. Sekigawa gives examples of such submanifolds for the three cases. Other interesting results were obtained by F. Dillen, B. Opozda, L. Verstraelen and L. Vrancken [**25**].

Many interesting results on AH-submanifolds of a NK-manifold are also obtained by V. F. Kirichenko [**66**].

Now we give some results on holomorphic manifolds of l.c.K-manifolds. A holomorphic submanifold of l.c.K-manifold inherits, necessarily, a l.c.K-structure. For g.H-manifolds we have a more precise result.

Theorem 7 [**108**]. *Let M be a holomorphic submanifold of a g.H-manifold $\widetilde{M}$. Then M has an induced g.H-manifold structure iff it satisfies one of the following two conditions:* a) *M is minimal,* b) *M is tangent to the Lee vector field of $\widetilde{M}$.*

We say that a g.H-manifold is a P_0K*-manifold* if it is locally conformal flat or, equivalently, the local Kähler metric of $\widetilde{M}$ is flat (cf. [107]).

Theorem 8 [56]. *A holomorphic submanifold tangent to the Lee vector field of a* P_0K*-manifold* $\widetilde{M}$ *is totally geodesic iff its normal bundle is flat, and it is locally symmetric iff it has parallel second fundamental form.*

For other interesting results on the holomorphic submanifolds of a l.c.K-manifold see S. Dragomir [30].

4 - Anti-holomorphic submanifolds of a AH-manifold

Let $(\widetilde{M}, J, g)$ be an AH-manifold. A submanifold M of $\widetilde{M}$ is called an *anti-holomorphic* (or *anti-invariant*, or, *totally real*), *submanifold* of $\widetilde{M}$ if $JT_pM \subset T_pM^{\perp}$ for each point p of M. If X is a tangent vector of M at p, then JX is a vector normal to M. Therefore we have $2 \dim M \leqslant \dim \widetilde{M}$.

There exist many anti-holomorphic submanifods of AH-manifolds. Reference books for the anti-holomorphic submanifods of a Kähler manifold are [117] and [16].

Let M be a submanifold of a generalized complex space form $\widetilde{M}(\mu, \alpha)$ which is not a manifold with constant sectional curvature ($\mu \neq \alpha$). It is easy to prove that M is a holomorphic or an anti-holomorphic submanifold of $\widetilde{M}(\mu, \alpha)$ iff M is an invariant submanifold (with respect to the curvature transformation of $\widetilde{M}$). This result was proved by B. Y. Chen and K. Ogiue for the submanifolds of complex space forms (cf. [19], [112]).

Now we give some results of L. Vanhecke.

Theorem 9 [112]. *Let M be an n-dimensional compact anti-holomorphic minimal submanifold immersed in $\widetilde{M}(\mu, \alpha)$ with* $\dim \widetilde{M} = 2n$ *and* $\nu > 0$. *If* $\|h\|^2 \leqslant \dfrac{n(n+1)}{2n-1}\nu$ *or equivalently* $\rho > \dfrac{2n^2(n-2)}{2n-1}\nu$, *then M is totally geodesic (Here $\nu = \mu + 3\alpha$ and ρ is the scalar curvature of M).*

Theorem 10 [115]. *Let M be an n-dimensional totally umbilical submanifold ($n > 2$) of an m-dimensional generalized complex space form $\widetilde{M}(\mu, \alpha)$ which is not a space of constant curvature. Then M is one of the following types:*

a) *a space of constant curvature* $\nu = \frac{1}{2}(\mu + 3\alpha)$ *immersed in* $\tilde{M}$ *as an anti-holomorphic and totally geodesic submanifold*

b) *a space of constant curvature* $\nu + g(H, H)$ *immersed in* $\tilde{M}$ *as an anti-holomorphic submanifold with nonzero parallel mean curvature vector*

c) *a generalized complex space form* $M(\mu, \alpha)$ *immersed in* $\tilde{M}$ *as a holomorphic and totally geodesic submanifold*

d) *a generalized complex space form* $M(\mu + g(H, H), \alpha + g(H, H))$ *immersed in* $\tilde{M}$ *as a holomorphic submanifold with nonzero parallel mean curvature vector (an extrinsic sphere of* $\tilde{M}$*).*

The case a) *occurs only when* $n \leqslant \frac{1}{2}m$ *and case* b) *only when* $n < \frac{1}{2}m$.

Concerning the anti-holomorphic submanifolds of a NK-manifold, M. Barros and J. Castellano proved the following result

Theorem 11 [3]. *Let M be an anti-holomorphic submanifold of a NK-manifold M. If M is totally umbilical and the f-structure in the normal bundle is parallel, then M is totally geodesic.*

We recall that on an anti-holomorphic submanifold M of $\tilde{M}$, we have $J\xi = t\xi + f\xi$ where $t\xi$ and $f\xi$ denote the tangent and the normal component of $J\xi$. Then f is an endomorphism of the normal bundle $TM^{\perp}$ and is an *f-structure* (cf. K. Yano and M. Kon [117]).

Some interesting results on the anti-holomorphic submanifolds of a the nearly Kähler 6-sphere have been obtained by N. E. Ejiri [35] and F. Dillen, B. Opozda, L. Verstraelen and L. Vrancken [24], [26].

Theorem 12 [35]. *A 3-dimensional anti-holomorphic submanifold of* S^6 *is orientable and minimal.*

Theorem 13 [35]. *Let M be a 3-dimensional anti-holomorphic submanifold of* S^6. *If M has constant sectional curvature K, then* $K=1$ *or* $K=\frac{1}{16}$.

Theorem 14 [24]. *If M is compact 3-dimensional anti-holomorphic submanifold of* S^6 *and the sectional curvature K of M satisfies* $\frac{1}{16} < K \leqslant 1$, *then* $K = 1$.

Theorem 15 [25]. *If M is minimal totally real surface of S^6 and M is homeomorphic to a sphere, then M is totally geodesic.*

Corollario 2 [25]. *If M is a compact minimal totally real surface of S^6 with Gaussian curvature K satisfying $0 \leqslant K \leqslant 1$, then either $K = 0$ or $K = 1$.*

It is easy to see that the exceptional simple Lie group G_2 operates transitively on the 6-dimensional unit sphere S^6. There exists essentially a unique G_2-invariant almost complex structure on S^6. K. Mashimo defines the four anti-holomorphic 3-dimensional compact submanifolds M_j of S^6 as orbits of certain subgroups L_j of G_2 ($j = 1, 2, 3, 4$).

Theorem 16 [73]. *Let M be a 3-dimensional anti-holomorphic submanifold of S^6 which is obtained as an orbit of a closed subgroup of G_2. Then M is congruent (by an element of G_2) to one of the M_1, M_2, M_3 or M_4 unless it is a standard sphere.*

Now we refer to some anti-holomorphic submanifolds of the l.c.K-manifolds. Let D^1 (resp. D^2) be the 1-dimensional distribution generated by the Lee vector field U (resp. V) on a g.H-manifold $\tilde{M}$ and $\dim \tilde{M} = 2m$. We denote by D^3 the complementary orthogonal distribution of $D^1 \oplus D^2$ (cf. [58]).

Proposition 1 [57], [63]. *Let M be an integral manifold of the distribution D^3. Then M is an anti-holomorphic submanifold of $\tilde{M}$ and* $\dim M \leqslant m - 1$.

There are examples in which the maximal dimension $m - 1$ can be effectively reached (cf. [57]).

Proposition 2 [57]. *Let M be an integral manifold of the distribution D^3. If the f-structure in the normal bundle is parallel, then M is totally geodesic.*

We say that M is an *extrinsic sphere of $\tilde{M}$ if M is a totally umbilical submanifold with nonzero parallel mean curvature vector field H* (cf. [16]).

Theorem 17 [57]. *Let $\tilde{M}$ be a g.H-manifold and M a complete, connected submanifold in $\tilde{M}$ with flat normal bundle. If M is anti-holomorphic submanifold of $\tilde{M}$ and normal to the Lee vector field U, then M is isometric to an ordinary sphere.*

Finally, we give a theorem by G. B. Rizza and the present author for anti-holomorphic extrinsic sphere in para-Kähler manifolds.

Theorem 18 [62]. *Let M be an anti-holomorphic extrinsic sphere of a para-Kähler manifold $\widetilde{M}$. If the normal bundle $TM^{\perp}$ is flat, then M is of constant sectional curvature $K = g(H, H)$.*

5 - CR-submanifolds of an AH-manifold

A *CR-submanifold* M is defined by A. Bejancu (see [5], [8]) as a real submanifold of an AH-manifold $\widetilde{M}(J, g)$ such that there exists a distrinbution D on M satistying the conditions $JD_p = D_p$ and $J(D_p^{\perp}) \subset T_p M^{\perp}$, for each $p \in M$, where $D^{\perp}$ is the complementary orthogonal distribution to D.

We denote by q the complex dimension of the distribution D and by r the real dimension of the distribution $D^{\perp}$. Then for $r = 0$ (resp. $q = 0$) a CR-submanifold becomes a holomorphic submanifold (resp. an anti-holomorphic submanifold). The standard references for examples and basic properties are the books of A. Bejancu [8]), and of K. Yano - M. Kon [118]. Also much valuable information is contained in the book of B. Y. Chen [16].

Many important results on CR-submanifolds of the Kähler manifolds have been obtained by many differential geometers in the last 15 years. In this section we refer to CR-submanifolds of an AH-manifold without Kähler condition.

Now we consider a riemannian manifold M isometrically immersed in an AH-manifold $\widetilde{M}(J, g)$. For each vector field X tangent to M we put

$$JX = \varphi X + \omega X \tag{5.1}$$

where φX and ωX are respectively the tangent part and the normal part of JX. A. Bejancu proved that M is a CR-submanifold in $\widetilde{M}$ iff rank φ is constant and $\omega \circ \varphi = 0$.

Now we recall the definition of a *Cauchy-Riemann manifold* (cf. Greenfield [54]). Let M be a differentiable manifold and $T^c M$ the complexifield tangent bundle to M. We say that M is Cauchy-Riemann manifold if there exists on M a complex subbundle H of $T^c M$ such that $H \cap \overline{H} = 0$ and H is involutive.

D. E. Blair and B. Y. Chen proved the property

Theorem 19 [9]. *A CR-submanifold of a Hermitian manifold is a Cauchy-Riemann manifold.*

They proved also that if M is a CR-submanifold of a l.c.K-manifold $\widetilde{M}$, then the anti-holomorphic distribution $D^{\perp}$ is always integrable. On the other hand, the integrability of both distributions D and $D^{\perp}$ has been studied by A. Bejancu when the ambient manifold $\widetilde{M}$ is an AH-manifold (cf. [6]).

N. Sato [94], [95] and F. Urbano [105] studied the integrability of distributions D and $D^{\perp}$ for a CR-submanifold of a nearly Kähler manifold. A CR-manifold of an AH-manifold is called a *mixed geodesic CR-submanifold* if its second fundamental form h satisfies (cf. [8])

$$h(X, U) = 0 \qquad \text{for any } X \in \Gamma(D) \text{ and } U \in \Gamma(D^{\perp}). \tag{5.2}$$

Proposition 3 [8]. *Let M be a CR-submanifold of an AH-manifold $\widetilde{M}$. Then M is mixed geodesic if and only if both distributions D and $D^{\perp}$ are invariant with respect to the action of Weingarten's operators, i.e.,*

$$A_V X \in \Gamma(D) \ \textit{and} \ A_V U \in \Gamma(D^{\perp}) \tag{5.2}$$

for any $X \in \Gamma(D)$, $U \in \Gamma(D^{\perp})$ and $V \in \Gamma(TM^{\perp})$.

A CR-submanifold M of an AH-manifold $\widetilde{M}$ is a *CR-product* if both distributions D and $D^{\perp}$ are integrable and M is locally a riemannian product $M_1 \times M_2$, where M_1 is a leaf of D and M_2 is a leaf of $D^{\perp}$.

N. Sato gives some sufficient conditions for a CR-submanifold M of a NK-manifold $\widetilde{M}$ to be a CR-product in $\widetilde{M}$ [95]. K. Sekigawa proved the following interesting result.

Theorem 20 [99]. *There does not exist any proper CR-product in S_6.*

However, K. Sekigawa gives an example of a minimal proper CR-submanifold of S_6 such that both distributions on it are integrable.

A CR-submanifold M of an AH-manifold $\widetilde{M}$ is said to be a *generic submanifold* if $\dim D_p^{\perp} = \dim T_pM^{\perp}$ for any $p \in M$ (cf. [118]). For some properties of generic submanifolds in AH-manifolds see A. Bejancu [7].

Some results for CR-submanifolds in a generalized complex space form are obtained by M. Barros and F. Urbano (see [4]).

In the last decade many researches have been done for the CR-submanifolds of l.c.K-manifolds by D. Blair, B. Y. Chen, S. Dragomir, K. Matsumoto, L. Ornea, P. Piccinni etc.

Theorem 21 [27]. *Let M be a mixed totally geodesic CR-submanifold of a l.c.K-manifold $\widetilde{M}$. If the Lee vector field of $\widetilde{M}$ is normal to M, then any maximal integral manifold of the totally real distribution is a totally geodesic submanifold of M.*

Theorem 22 [27]. *Any proper CR-product in a complex Hopf manifold is a mixed totally geodesic submanifold.*

Theorem 23 [31]. *Let M be a geodesic CR-submanifold of a l.c.K-manifold $\tilde{M}$. If the holomorphic distribution D of M is completely integrable and its leaves are totally geodesic, then the leaves are totally umbilical in $\tilde{M}$.*

This result generalizes a result by B. Y. Chen for CR-submanifolds of a Kähler manifold (see [15]).

Now, let V be a real $2n$-dimensional vector space with a hermitian structure (g, j) and let W be an oriented $2p$-dimensional subspace of V. We denote by Σ_W the system of the $2p$-dimensional holomorphic spaces of V $(1 \leq p \leq n)$ having a p-dimensional anti-holomorphic space in common with W. G. B. Rizza proved that the space W forms a constant angle δ_W $(0 \leq \delta_W \leq \pi)$ with the holomorphic spaces of Σ_W (see [88], [89]). The angle δ_W is called the *holomorphic deviation of* W.

In the paper [61] G. B. Rizza and the present author consider the special class, denoted by $\mathcal{D}$, of the even-dimensional submanifolds of an AH-manifold for which the tangent spaces have a constant holomorphic deviation. Obivously, the class $\mathcal{D}$ contains the holomorphic and anti-holomorphic submanifolds.

Theorem 24 [61]. *Let δ be a real number satisfying $0 \leq \delta \leq \pi$. Then in C^n $(n \geq 2)$ there exists a $2p$-dimensional submanifold M $(p = 1, \ldots, n-1)$ having constant holomorphic deviation δ.*

Let W', W'' be oriented r-dimensional vector spaces of V. We say that W' is *orthogonal* to W'' iff there exists a non zero vector of V' which is orthogonal to any vector of W'' (and symmetrically). Let $(X_1, \ldots, X_r)$, $(Y_1, \ldots, Y_r)$ be oriented bases in W' and W''. Then W' is orthogonal to W'' iff it satisfies

$$\det(g(X_i, Y_j)) = 0 . \tag{5.3}$$

A subspace W of V is called *weakly anti-holomorphic* iff W and JW are orthogonal. If W is weakly anti-holomorphic, then $\delta_W = \frac{\pi}{2}$ and conversely. (cf. [61]).

A submanifold M of an AH-manifold $\tilde{M}$ is called a *weakly anti-holomorphic submanifold* of $\tilde{M}$ [61], if

$$\delta_{T_pM} = \frac{\pi}{2} \qquad \text{for any } p \in M . \tag{5.4}$$

If there exists on M a never vanishing vector field ξ, such that $(J\xi)_p \in T_pM^{\perp}$ for any point $p \in M$, then M is a weakly anti-holomorphic submanifold.

Any nonholomorphic CR-submanifold M of an AH-manifold $\widetilde{M}$ is a weakly anti-holomorphic submanifold.

Recently, B. Y. Chen introduced the concept of slant submanifold. Let M a submanifold of an AH-manifold $\widetilde{M}$. For any nonzero vector X tangent to M at a point $p \in M$ let $\theta(X)$ be the angle between JX and the tangent space T_pM. M is called a *slant submanifold* if the angle $\theta(X)$ is constant (i.e. $\theta(X)$ is independent of the choice of $p \in M$ and of $X \in T_pM$) (see [**17**]). The notion of oriented slant surface and of surface having constant holomorphic deviation coincide, but for higher dimensions the two concepts are different [**61**].

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