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On the topology of Landsberg spaces (**)

1 - The main results

Let M be a Landsberg space, $\dim(M) = n$, $n \geqq 3$. Let E: $T(M) \to M$ denote its Finsler energy; let π: $V(M) \to M$ be the natural projection, $V(M) = T(M) \setminus 0$. Let $\pi^{-1} T(M) \to V(M)$ be the pullback of the tangent bundle $T(M)$ by π. We denote by g the Riemann bundle metric of $\pi^{-1} T(M)$ induced by E, i.e. $g(X_i, X_j) = g_{ij}$, where

$g_{ij} = \frac{1}{2} \dot{\partial}_i \dot{\partial}_j E$, $X_i(u) = (u, \partial_i|_{\pi(u)})$, $u \in V(M)$ $\dot{\partial}_i = \partial / \partial y^i$ $\partial_i = \partial / \partial x^i$.

Let ∇ be the Cartan connection of (M, E) and N its non-linear connection on $V(M)$; let β: $\pi^{-1} T(M) \to N$ be the corresponding horizontal lift, i.e. $\beta X_i = \delta_i$ $(i = 1, 2, \ldots, n)$. Here $\delta_i = \partial_i - N_i^j \dot{\partial}_j$. Also N_j^i are given by the formula (18.15) in ref. $[9]_1$ (p. 118). Denote by γ: $\pi^{-1} T(M) \to \mathrm{Ker}(d\pi)$ the mapping defined as follows: if $X \in \pi^{-1} T(M)$, $X = (u, v)$, then $\gamma X = \frac{dc}{dt}(0)$, where c: $[0, 1] \to V(M)$ is the curve such that $c(t) = u + tv$, $t \in [0, 1]$. The Sasaki lift of g to $V(M)$ is the Riemann metric

$$\tilde{g}(Z, W) = g(LZ, LW) + g(GZ, GW) \tag{1.1}$$

where $L \partial_i = X_i$, $L \dot{\partial}_i = 0$, $G \delta_i = 0$, $G \dot{\partial}_i = X_i$. Let $R^1(X_j, X_k) = R^i_{jk} X_i$, $C(X_j, X_k) = C^i_{jk} X_i$, where $R^i_{jk} = \delta_j N^i_k - \delta_k N^i_j$, while C^i_{jk} are given by (17.1) in ref.

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[9]$_1$ (p. 109). Let R^i_{jkm}, S^i_{jkm} be the horizontal and vertical curvature tensors of ∇, respectively: we put $r(X_j, X_k) = R_{jk}$, $s(X_j, X_k) = S_{jk}$, where $R_{jk} = R^i_{ijk}$, $S_{jk} = S^i_{ijk}$. We shall also need the exterior v-differentiation operator d^v (cf. Z. I. Szabo, [16], p. 165). Let us consider the following inequalities

$$(1.2)\qquad r(X, X) + \mathrm{Trace}\{Y \to C(R^1(X, Y), X)\}$$
$$\geqq (\gamma C(X, X)) \log \sqrt{g} + \frac{1}{2} g^{ij} g(R^1(X, X_i), R^1(X, X_j))$$

$$(1.3)\qquad g^{ij} g(X, (\nabla_{\delta_i} R^1)(Y, X_j)) \geqq 0$$

$$(1.4)\qquad s(X, X) \geqq (\nabla_{\gamma X} d^v \log \sqrt{g}) X + \mathrm{Trace}\{Y \to C(C(X, Y), X)\}$$
$$- \frac{1}{4} g^{ij} g^{km} g(R^1(X_i, X_k), X) g(R^1(X_j, X_m), X) + \frac{1}{2} g^{ij} g(C(R^1(X, X_i), X_j), X)$$

where $g_{ij} g^{jk} = \delta_i^k$. We obtain the following

Theorem 1. *Let M be a n-dimensional Landsberg space, $n \geqq 3$. If the Sasakian lift of the Landsberg metric is complete and* (1.2)-(1.4) *are verified for any Finsler vector fields X, Y on M, then the growth function of any finitely generated subgroup H of the first homotopy group $\pi_1(M)$ is subject to*

$$(1.5)\qquad gr_H(s) \leqq \frac{\omega_{2n}(\mu + \varepsilon)^{2n}}{v(\varepsilon)} s^{2n} .$$

Here ω_{2n} is the volume of the unit ball in $\mathbb{R}^{2n}$, while $v(r)$ denotes the volume of the compact ball of radius $r > 0$ with respect to the induced metric on the universal covering manifold of $V(M)$.

Theorem 2. *Let M^2 be a totally geodesic closed Landsberg surface of a Finsler space M^{n+2} of positive scalar curvature. Then M^2 has a non vanishing Euler-Poincaré characteristic.*

Note that Theorem 1 extends a result of J. Milnor [10] to the case of Landsberg spaces; it might be contrasted with the Finslerian version of S. Myers' Theorem (cf. F. Moalla [11]).

The Theorem 2 is based mainly on the Gauss-Bonnet formula for Landsberg spaces, such as obtained by A. Lichnerowicz [8] and P. Dazord [4].

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2 - Proof of Theorem 1

We firstly show that *the fundamental groups* $\pi_1(V(M))$ *and* $\pi_1(M)$ *are isomorphic*. Indeed, if M is non-compact, then by Corollary III (p. 399) in [6], there exists a globally defined everywhere non-vanishing vector field on M (a Landsberg space is supposed to be connected), i.e. a globally defined cross-section in $V(M)$. Consequently, by a theorem of N. Steenrod [14] (i.e. Th. 17.7, p. 92), if such a cross-section exists, then $\pi_1(V(M)) = G_1 \cdot G_2$, where $G_1 \approx \pi_1(M)$, $G_2 \approx \pi_1(V(x_0))$, $V(x_0) = \pi^{-1}(x_0)$, for a given base point $x_0 \in M$. Since $V(x_0) \approx R^n_* = R^n \setminus \{0\}$ and $n > 2$, we have $\pi_1(V(x_0)) = 0$. This proves $\pi_1(M) \approx \pi_1(V(M))$. If, in turn, M is compact then there exists a globally defined everywhere non-vanishing vector on M if and only if $\chi(M) = 0$, i.e. the Euler-Poincaré characteristic of M is vanishing. So generally we cannot use again the above N. Steenrod's theorem. However, if M is compact, then (M, E) is complete (cf. B. T. Hassan [7]); then the induced group homomorphism $\pi_*: \pi_1(V(M)) \to \pi_1(M)$ is an isomorphism by the following argumentation. We firstly show that π_* is a monomorphism. To this end we consider the homotopy sequence of the fibre bundle $\pi: V(M) \to M$ i.e.

$$(2.1) \qquad \ldots \xrightarrow{\pi_*} \pi_2(M) \xrightarrow{\Delta} \pi_1(V(x_0)) \xrightarrow{i_*} \pi_1(V(M)) \xrightarrow{\pi_*} \pi_1(M) \,.$$

But (2.1) is exact and $\pi_1(V(x_0)) = 0$, $n > 2$, so that the sequence $0 \to \pi_1(V(M)) \to \pi_1(M)$ is exact.

Next we show that π_* is an epimorphism. Let $h \in \pi_1(M)$, $h \neq l$. Let $x_0 \in M$ be a fixed base point and l''_{x_0} a loop at x_0 representing the class h, i.e. $h = \langle l''_{x_0} \rangle$. Let $l_{x_0} \in h$ be chosen such that $L(l_{x_0}) = \inf\{L(l''_{x_0}) \mid l''_{x_0} \in h\}$; here $L(c)$ denotes the (Finslerian) length of the curve c. Then l_{x_0} is a geodesic of (M, E) (see also J. Synge [15]). Indeed, since the property of being a geodesic is local, it suffices to show that l_{x_0} is a geodesic between any two points (on l_{x_0}) lying in a simply-connected neighbourhood in M. Suppose that, on the contrary, there exist two values of the parameter, say $t_1 < t_2$, such that $p = l_{x_0}(t_1)$, $q = l_{x_0}(t_2)$, $p \neq q$, and the portion of l_{x_0} between p, q is not a geodesic and $p, q \in U$, $\pi_1(U) = 0$. Let c be the geodesic connecting p and q. Such a geodesic always exists since (M, E) is complete. Let l'_{x_0} be the loop at x_0 obtained by joining the portion of l_{x_0} between x_0 and p with c followed by the portion of l_{x_0} between q and x_0, for a fixed orientation of l_{x_0}. Since $\pi_1(U) = 0$, l_{x_0} and l'_{x_0} are clearly homotopic. Thus $l'_{x_0} \in h$. This is a contradiction as $L(l'_{x_0}) < L(l_{x_0})$, due to the inferior length of l'_{x_0} on its portion c. Since l_{x_0} is a geodesic, it is in particular regular; therefore, its natural lift $L'_{x_0}(t) = \dfrac{dl_{x_0}}{dt}(t)$, $t \in [0, 1]$, is a curve in $V(M)$. If u_0 denotes the tangent vector of l_{x_0} at x_0, then $L'_{x_0}(0) = u_0$ and $u_0 \in V(x_0)$. Obviously the end point $u'_0 = L'_{x_0}(1)$ of L'_{x_0} still belongs to the fibre $V(x_0)$, yet generally $u_0 \neq u'_0$, so L'_{x_0} is not a loop at u_0. However, since $V(x_0) \approx \mathbb{R}^n_*$, $V(x_0)$ is pathwise connected. Let then $c\colon [0, 1] \to V(x_0)$ be a curve such that $c(0) = u'_0$, $c(1) = u_0$. We construct the following loop at u_0: $L_{u_0}(t) = L'_{x_0}(2t)$, $0 \leqq t \leqq 1/2$ and $L_{u_0}(t) = c(2t-1)$, $1/2 \leqq t \leqq 1$. Let l'_{x_0} be the projection of L_{u_0} on M, i.e. $l'_{x_0} = \pi \cdot L_{u_0}$. Then l_{x_0} and l'_{x_0} are homotopic, by the homotopy $H(s, t) = x_0$ iff $t \geqq 1 - s/2$ and $H(s, t) = l_{x_0}(t/(1-s/2))$ iff $0 \leqq t \leqq 1 - s/2$. Consequently $\pi_*\langle L_{x_0}\rangle = h$ and π_* is an epimorphism.

Let H be a finitely generated subgroup of $\pi_1(M)$ and $h_1, \ldots, h_p$ a system of generators of H. Then the growth function of H (associated with $h_1, \ldots, h_p$) is given by

$$gr_H(s) = \operatorname{card}\{\prod_{i=1}^{p} h_i^{m_i} \mid \sum_{i=1}^{p} |m_i| \leqq s,\ m_i \in \mathbb{Z}\} \qquad s \in \mathbb{Z} \qquad s > 0\,.$$

Let $g_i \in \pi_1(V(M))$ $(i = 1, \ldots, p)$ corresponding to h_i by the isomorphism π_*. Let $\widetilde{V(M)}$ be the universal covering manifold of $V(M)$ and $p\colon \widetilde{V(M)} \to V(M)$ the natural projection. The $p^*\tilde{g}$ is a Riemann metric on $\widetilde{V(M)}$, associated with (1.1); let D be the corresponding distance function. Moreover, let $v_0 \in \widetilde{V(M)}$ be fixed and let $v(r) = \operatorname{vol}(N_r(v_0))$, where $N_r(v_0) = \{v \in \widetilde{V(M)} \mid D(v_0, v) \leqq r\}$, $r > 0$. Since we have (1.2)-(1.4), the mean curvature of (1.1) is non-negative (cf. our n. 3). Now we apply the proof of J. Milnor (op. cit.) to establish the constant in the inequality (1.5); that is, by a result of L. Bishop [1] we have

$$v(r) \leqq \omega_{2n} r^{2n}\,. \tag{2.2}$$

Think of elements of $\pi_1(V(M))$ as deck transformations of $\widetilde{V(M)}$. Let $\mu = \max_{i=1}^{p} D(v_0, g_i(v_0))$, $\mu > 0$. Then $g_i(v_0) \in N_\mu(v_0)$ $(i = 1, \dots, p)$. Any deck transformation is a D-isometry, such that

$$
\begin{aligned}
&D(v_0, h^m(v_0)) \leqq |m| D(v_0, h(v_0)) \\
&D(v_0, (hh')(v_0)) \leqq D(v_0, hv_0) + D(v_0, h' v_0)
\end{aligned}
\tag{2.3}
$$

for any $h, h' \in \mathcal{H}$, $m \in \mathbb{Z}$. Here $\mathcal{H}$ is (uniquely) determined by $\pi_* \mathcal{H} = H$. Let $g = \prod_{i=1}^{p} g_i^{m_i}$ be a word in $\mathcal{H}$ of length at most s. By (2.3): $D(v_0, g(v_0)) \leqq \mu s$. Consequently $\operatorname{card}\{g(v_0) \in N_{\mu s}(v_0) \mid g \in \mathcal{H}\} \geqq \operatorname{gr}_{\mathcal{H}}(s)$. Since $\pi_1(V(M))$ acts properly discontinuously on $\widetilde{V(M)}$, we may choose $\varepsilon > 0$ such that $N_\varepsilon(v_0) \cap g(N_\varepsilon(v_0)) = \emptyset$ whenever $g \neq 1$. Then $N_{\mu s + \varepsilon}(v_0)$ contains at least $\operatorname{gr}_{\mathcal{H}}(s) = \operatorname{gr}_H(s)$ disjoint sets of the form $g(N_\varepsilon(v_0))$ and we get $\operatorname{gr}_H(s)\, v(\varepsilon) \leqq v(\mu s + \varepsilon)$; using this and (2.2) we obtain (1.5).

We wish to underline that the difficulties in the above proof are furnished by the lack of differentiability of the Finsler energy function E (only of class C^1 along the zero section in $T(M)$). This prompts our choice of bundle $(V(M), \pi, M, \mathbb{R}^n_*)$, rather than the whole of $T(M)$ (where we automatically have $\pi_1(T(M)) \approx \pi_1(M)$, since $T(M)$ admits globally defined cross-sections).

3 - The curvature of the Sasaki metric

Next we discuss the conditions (1.2)-(1.4). T. E. Davies-K. Yano [4], [5] have considered the following linear connection on $V(M)$, (associated with the Cartan connection)

$$\tilde{\nabla}_Z W = \beta \nabla_Z LW + \gamma \nabla_Z GW . \tag{3.1}$$

Note that $\tilde{\nabla} \tilde{g} = 0$, but ∇ has non-trivial torson form, say A. If M is a Landsberg space, i.e. the Berwald connection of M is h-metrical (cf. also Th. 25.3 in $[9]_1$, p. 162), then

$$A(\beta X, \beta Y) = \gamma R^1(X, Y) \qquad A(\gamma X, \beta Y) = \beta C(X, Y) \qquad A(\gamma X, \gamma Y) = 0 . \tag{3.2}$$

One may use (3.1) to determine the Levi-Civita connection of $(V(M), \tilde{g})$, say D. That is, we have

$$2\tilde{g}(D_X Y, Z) = 2\tilde{g}(\tilde{\nabla}_X Y, Z) + \tilde{g}(A(Y, Z), X) + \tilde{g}(A(X, Z), Y) + \tilde{g}(A(Y, X), Z)$$

for any tangent vector fields X, Y, Z on $V(M)$. Then (3.2) leads to

$$\tag{3.3} \begin{gathered} D_{\gamma X}\gamma Y = \gamma\nabla_{\gamma X} Y \qquad D_{\beta X}\beta Y = \beta\nabla_{\beta X} Y - \gamma\{\frac{1}{2}R^1(X, Y) + C(X, Y)\} \\ D_{\beta X}\gamma Y = \gamma\Delta_{\beta X} Y + \beta\{\frac{1}{2}(g(R^1(X, \cdot), Y))^{\#} + C(X, Y)\} \\ D_{\gamma X}\beta Y = \beta\{\frac{1}{2}(g(R^1(Y, \cdot), X))^{\#} + C(X, Y)\} \end{gathered}$$

for any Finsler vector fields X, Y on M. Here # denotes raising of indices by g. Straighforward computation based on (3.3) yields the expression of the Ricci curvature of (1.1)

$$\mathrm{Ric}(\dot{\partial}_j, \delta_k) X^j Y^k = \frac{1}{2} g^{ij} g(X, (\nabla_{\delta_i} R^1)(Y, X_j))$$

$$\mathrm{Ric}(\delta_j, \delta_k) X^j X^k = r(X, X) - (\gamma C(X, X)) \log\sqrt{g} + \mathrm{Trace}\{Y \rightarrow C(R^1(X, Y), X)\} - \frac{1}{2} g^{ij} g(R^1(X, X_i), R^1(X, X_j))$$

$$\mathrm{Ric}(\dot{\partial}_j, \dot{\partial}_k) X^j X^k = s(X, X) - \mathrm{Trace}\{Y \rightarrow C(X, C(Y, X))\} - (\nabla_{\gamma X} d^v \log\sqrt{g}) X - \frac{1}{4} g^{ij} g^{kl} g(R^1(X_i, X_k), X) g(R^1(X_j, X_l), X) - \frac{1}{2} g^{ij} g(R^1(C(X, X_i), X_j), X) .$$

Thus (1.2)-(1.4) are equivalent to $\mathrm{Ric}(Z, Z) \geqq 0$ for any tangent vector field Z on $V(M)$.

4 - Proof of Theorem 2

Let M^2 be a closed Landsberg surface of the Finsler space M^{n+2} of scalar curvature $K > 0$, $K \in C^\infty(V(M^{n+2}))$. We need the following Gauss-Codazzi equation (cf. [5], p. 6)

$$
\begin{aligned}
(4.1) \qquad & R^*(X,\ Y)Z + P^*(H(X,\ \bar{v}),\ Y)Z \\
& - P^*(H(Y,\ \bar{v}),\ X)Z + S^*(H(X,\ \bar{v}),\ H(Y,\ \bar{v}))Z \\
& = R(X,\ Y)Z + A_{H(X,Z)}Y - A_{H(Y,Z)}X + (D_{\beta X}H)(Y,\ Z) - (D_{\beta Y}H)(X,\ Z) \\
& \quad + H(T(X,\ Y),\ Z) + Q(R^1(X,\ Y),\ Z)\,.
\end{aligned}
$$

By direct extension of a result of M. G. Brown ([2], th. 6.2, p. 1035) to the arbitrary codimension case, M^2 is totally geodesic iff $N_0 = 0$, $N_0 = H(\bar{v},\ \bar{v})$. Moreover, as observed by O. Varga [17] $N_0 = 0$ iff $H(X,\ \bar{v})$ for any Finsler vector field X on M^2. Let L be the induced fundamental Finsler metric function on M^2, $L = E^{1/2}$. Let $U(M^2) \to M^2$ be the tangent sphere bundle of M^2; then $U(M^2)$ is a 3-dimensional hypersurface of the Riemann space $V(M^2)$ (carrying the Sasaki metric). Let l be the Finsler l-form given by $l = (\mathrm{d}L)\circ\gamma$. Since $U(M^2)$ is compact $\|l\|$ is bounded, i.e. $\|l\|_u \leqq A$, for any $u \in U(M^2)$ and some $A > 0$. Let $l_A = (n+2)^{1/2}A^{-1}l$. We also define $h_A = g - l_A \otimes l_A$. Moreover, M^{n+2} is said to be a Finsler space of *scalar curvature* K if for any Finsler vector fields X, Y, Z on M^{n+2} we have

$$
(4.2) \qquad g^*(R^{\perp *}(X,\ Y),\ Z) = \omega(Y)\,h_A(X,\ Z) - \omega(X)\,h_A(Y,\ Z)
$$

where $\omega = \dfrac{L^2}{3}\mathrm{d}^v K + KLl$. Note that our notation of Finsler space of scalar curvature differs slightly from that in $[9]_1$ (p. 168) where $A = (n+2)^{1/2}$.

Let (u^α) be a local coordinate system on M^2 and $(u^\alpha,\ v^\alpha)$ the induced coordinates on $V(M^2)$. Under the hypothesis of Th. 2, (4.1)-(4.2) lead to

$$
(4.3) \qquad R_{\alpha\sigma}v^\alpha v^\sigma = (n + 2 - \|l_A\|^2)KL^2\,.
$$

Here $R_{\alpha\sigma} = R^\lambda_{\lambda\alpha\sigma}$. Suppose now $\chi(M^2) = 0$, where $\chi(M^2)$ denotes the Euler-Poincaré characteristic of M^2; at this point we may use the Gauss-Bonnet formula [4], [8]

$$
\int_{U(M^2)} (R_{\alpha\sigma}v^\alpha v^\sigma)\,{*1} = 4\pi^2\chi(M^2)
$$

and $K > 0$, $n + 2 - \|l_A\|^2 > 0$, to obtain $L = 0$, a contradiction, since $\det(g_{\alpha\sigma}) \neq 0$.

Here $*1$ denotes the canonical Riemannian measure (associated with the induced metric) on the hypersurface $U(M^2)$. It is tempting to assume that M^{n+2} is itself Landsberg; then M^2 would inherit the Landsberg property, as a totally geodesic surface (cf. M. Matsumoto $[9]_2$). Yet, by a result of S. Numata [12] if $n \geqq 1$, M^{n+2} falls into nothing but a Riemann space-form.

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Riassunto

Si determina una limitazione per la funzione di accrescimento di un qualsiasi sottogruppo finitamente generato del gruppo fondamentale di uno spazio di Landsberg. Inoltre si prova che certe superfici di uno spazio di Finsler a curvatura scalare hanno caratteristica di Eulero-Poincaré non nulla.

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