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Relations between convergence, almost convergence and r-convergence (**)

1. - Introduction.

The Authors [3] introduced the notion of almost convergence of a filter base and obtained a filter characterisation of almost-regular spaces introduced [4] by Signal and Arya. Recently Herrington and Long [2] also defined a new type of convergence called « r-convergence » of a filter base in order to characterise H-closed spaces. In the present paper we determine the relationship between convergence, almost convergence and r-convergence of a filter base.

2. - Definitions and notations.

Let A be a subset of a topological space X. We shall denote the closure of A and the interior of A in X by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$ respectively. A is said to be *regular-open* if $A = \mathrm{Int}(\mathrm{Cl}(A))$.

Definition 2.1 ([5], p. 78). A filter base β *converges* to $x \in X$ iff each $N \in N(x)$, the neighborhood filter at x, contains some $B \in \beta$.

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Definition 2.2 [3]. Let β be a filter base in a topological space X. Then β is said to *almost converge* to a point $x \in X$, if β is a refinement of the filter base $N^*(x)$ of all regular-open neighborhoods of the point x.

Definition 2.3 [2]. The filter base $F = \{A_\alpha | \alpha \in A\}$ in the topological space X *r-converges to* $x_0 \in X$, if for each open set V containing x_0, there exists and $A_\alpha \in F$ such that $A_\alpha \subset \mathrm{Cl}(V)$.

Definition 2.4 [4]. A space X is *almost-regular* if for each point $x \in X$ and each regular-open set V containing x, there exists a regular-open set U such that $x \in U \subset \mathrm{Cl}(U) \subset V$.

Definition 2.5 ([1], p. 83). A space X is called a T_3^s*-space* if its regular-open sets form a base for the open sets of X.

3. - Convergence and almost convergence.

From Definition 2.1 and Definition 2.2, it follows that convergence implies almost convergence. The converse may not be true is shown by the following.

Example 3.1. Let N be the set of positive integers and R, the reals with the co-countable topology. Define a filter base β in R by

$$\beta = \{B_n | B_n = \{n, n+1, n+2, \ldots\}, n \in N\} .$$

Let $x_0 \in R$, and $U = (R - B_1) \cup \{x_0\}$. Then U is an open set which contains the point x_0 and so is a neighbourhood of x_0. But, since there does not exist a $B_n \in \beta$ such that $B_n \subset U$, it follows that β does not converge to the point x_0. However, since the interior of closure of any open set containing x_0 is R, β almost converges to x_0.

We now prove the following

Theorem 3.1. *Let X be a T_3^s-space. If a filter base β in X almost converges to a point $x \in X$, then β converges to x.*

Proof. Let N be any neighbourhood of the point x in X. Then there exists an open set U in X such that $x \in U \subset N$. Since X is a T_3^s-space, there exists a regular-open set N^* in X such that $x \in N^* \subset U \subset N$. Now almost convergence of β to x implies the existence of some $B \in \beta$ such that $B \subset N^*$. Thus we have a $B \in \beta$ such that $B \subset N$. Since N is any neighbourhood of the point x, it follows that β converges to x. Hence the theorem.

4. - Almost convergence and r-convergence.

It can be easily shown that almost convergence of a filter base in a topological space to a point implies its r-convergence to that point. The converse may not be true is shown by the following

Example 4.1. Let (X, T) be a topological space where $X = \{a, b, c\}$ and $T = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. Define a filter base β in X by $\beta = \{A \subset X \mid c \in A\}$. Consider the point $a \in X$. Since the closure of any open set containing a belongs to β, it follows that β r-converges to a. Again the set $\{a\}$ is a regular-open neighbourhood of the point a. Since no member of β is contained in $\{a\}$, it follows that β does not almost converge to the point a.

However we have the following

Theorem 4.1. *Let X be an almost-regular space. If a filter base β in X r-converges to a point $x \in X$, then β almost converges to x.*

Proof. Let N^* be any regular-open neighbourhood of the point $x \in X$. Since X is almost-regular, there exists a regular-open set U such that $x \in U \subset \mathrm{Cl}(U) \subset N^*$. Since U is a regular-open set and hence an open set containing the point x, and β r-converges to x, it follows that there exists $B \in \beta$ such that $B \subset \mathrm{Cl}(U)$, and therefore $B \subset N^*$. But N^* is arbitrary, it follows that β almost converges to x. Hence the theorem.

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Summary

See Introduction.

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