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A rigorous derivation of the reactor kinetics equation from the Chapman - Kolmogorov system. (**)

1. - Introduction.

In a recent paper [1], the Chapman-Kolmogorov system for neutron population in a multiplying assembly at zero power was studied by using the theory of semigroups. Existence and uniqueness was proved of a positive and norm-invariant solution belonging to the Banach space of summable sequences. Moreover, a procedure was indicated to derive rigorously the equation for the first moment of the neutron population.

In Sect. 2 of this paper, we summarize some basic results obtained in [1]. Sect. 3 is devoted to reformulate the problems and the procedures sketched in Sect. 7 of [1] in a simpler and more compact way. Finally, in Sect. 4 and 5, we present in detail a rigorous derivation of the equation for the first moment from the Chapman-Kolmogorov system. Such a derivation involves the study of a suitable « approximate » solution whose properties can be profitably used to obtain the corresponding properties of the « exact » solution (see Sect. 4).

Following [2] (see also refs. [3] to [11]), the Chapman-Kolmogorov system under consideration has the form

$$\frac{\partial}{\partial t} P(n, t) = -pnP(n, t) + p \sum_{s=0}^{n} b(s)(n+1-s)P(n+1-s, t) + \\ + q[P(n-1, t) - P(n, t)] , \qquad t > 0 , \ n = 0, 1, \ldots ,$$

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where:

$P(-1, t) = 0$;

$P(n, t) =$ the probability that n neutrons are in the multiplying assembly at the instant t;

$p = 1/l$, where l is the average lifetime;

$b(s)$, $s = 0, 1, \dots, =$ the probability that s neutrons are emitted if one neutron is absorbed;

$0 \leqslant b(s) \leqslant 1, \; \sum_{s=0}^{\infty} b(s) = 1$;

$q =$ the probability per unit time interval that a non-fission source emits a neutron.

The preceding system must be supplemented with a initial condition of the form

$$P(n, 0) = P_0(n)\,, \qquad n = 0, 1, \dots\,, \qquad \text{where} \qquad 0 \leqslant P_0(n) \leqslant 1\,, \qquad \sum_{n=0}^{\infty} P_0(n) = 1\,.$$

2. - Mathematical setting.

Following [1], let X be the Banach space of all summable sequences of real numbers

$$X = \{f\colon f = \{f(n), n = 0, 1, \dots\}, \; \|f\| = \sum_{n=0}^{\infty} |f(n)| < \infty\}$$

and let X_+ be the positive cone of X

$$X_+ = \{f\colon f \in X;\; f(n) \geqslant 0, n = 0, 1, \dots\}\,.$$

Let us also define the following operators

$$(1) \quad \begin{cases} [Af]_n = -pnf(n) + p \sum_{s=0}^{n} b(s)(n+1-s)f(n+1-s) \qquad (n = 0, 1, \dots) \\ D(A) = \{f\colon f \in X;\; \sum_{n=0}^{\infty} |[Af]_n| < \infty\}\,; \end{cases}$$

$$\text{(2)} \qquad [Hf]_n = pnf(n) \quad (n = 0, 1, \dots), \quad D(H) = D = \{f\colon f \in X;\ \sum_{n=0}^{\infty} n\,|f(n)| < \infty\},$$

$$\text{(3)} \qquad [Kf]_n = p\sum_{s=0}^{n} b(s)(n+1-s)f(n+1-s) \quad (n=0, 1, \dots), D(K)=D(H)= D,$$

$$\text{(4)} \qquad [Sf]_n = qf(n-1) \qquad (n = 1, 2, \dots), \qquad [Sf]_0 = 0, \qquad D(S) = X,$$

where $[Af]_n$ indicates the $(n+1)$-th component of Af and where $D(A)$ is the domain of A.

By using definitions (1) and (4), the abstract version of the stochastic model for neutron multiplication of Sect. 1 can be written as follows

$$\text{(5)} \qquad \frac{d}{dt}u(t) = (A + S - qI)u(t), \quad t > 0; \qquad \lim_{t\to 0_+} \|u(t) - u_0\| = 0.$$

In system (5), $u(t) = \{P(n, t), n = 0, 1, \dots\}$ is a map from $[0, +\infty)$ into X, d/dt is a strong derivative [**12**], and $u_0 = \{u_0(n) = P_0(n), n = 0, 1, \dots\}$.

We have [**1**], [**12**]

Lemma 1. *Let*

$$\text{(6)} \qquad G_r = -H + rK, \qquad D(G_r) = D,$$

where r is a real parameter, such that $0 \leqslant r < 1$. Then, $G_r \in \mathscr{G}(1, 0)$, ([**12**]$_2$, p. 485) *and the semigroup $Z_r(t) = \exp(tG_r)$ maps X_+ into itself.*

Moreover, let

$$\text{(7)} \qquad Z(t)f = \lim_{r\to 1_-} Z_r(t)f, \qquad f \in X,\ t \geqslant 0.$$

Then:

(*a*) *relation* (7) *holds uniformly with respect to t in each finite interval $[0, \bar{t}]$;*
(*b*) *$Z(t)$ is a semigroup such that $\|Z(t)\| \leqslant 1$ and $Z(t)[X_+] \subset X_+$ at any $t \geqslant 0$;*
(*c*) *if G is the generator of $Z(t)$, then $G \in \mathscr{G}(1, 0)$ and $(-H + K) \subset G \subset A$;*
(*d*) $\|Z(t)f\| = \|\exp(tG)f\| = \|f\|$, $\forall f \in X_+$, $t \geqslant 0$.

Lemma 2. $S[X_+] \subset X_+$, $\|Sf\| = q\|f\|$, $\forall f \in X_+$, $\|S\| = q$. *As a consequence*

$$\text{(8)} \qquad \chi = G + S - qI \in \mathscr{G}(1, 0),$$

$$\|\exp(t\chi)f\| = \|f\|, \qquad \forall f \in X_+,\ t \geqslant 0, \tag{9}$$

where the semigroup $\exp(t\chi) = \exp(-qt)\exp[t(G+S)]$ *maps* X_+ *into itself.* ∎

Lemma 3. *The initial-value problem*

$$\frac{d}{dt}u(t) = \chi u(t) \quad (t>0), \qquad \lim_{t\to 0_+} u(t) = u_0 \tag{10}$$

admits a unique continuous and differentiable solution

$$u(t) = \exp(t\chi)u_0 \qquad (t \geqslant 0), \tag{11}$$

provided that $u_0 \in D(G) = D(\chi)$. *Moreover,*

$$u(t) \in D_+(G) = D(G) \cap X_+, \qquad \|u(t)\| = 1, \quad \forall t \geqslant 0 \tag{12}$$

if $u_0 \in D_+(G)$ *and* $\|u_0\| = 1$. ∎

Remark 1. The preceding lemmas summarize most of the results obtained in [**1**] by using lemmas and theorems of [**12**].

Remark 2. Due to (*c*) of Lemma 1, $D = D(-H+K) \subset D(G) \subset D(A)$. Hence, $u(t) \in D(A)$, $Au(t) = Gu(t)$ at any $t \geqslant 0$ and the $u(t)$ given by (11) also satisfies the « physical » system (5). We also note that the assumptions of Lemma 3 are satisfied if in particular $u_0 \in D_+ = D \cap X_+$ and $\|u_0\| = 1$.

3. - Further preliminary remarks.

If we multiply both sides of the $(n+2)$-th component of the two (5) by $(n+1)$ and if we put

$$v(n,t) = (n+1)\,u(n+1,t) \qquad (n = 0, 1, \ldots), \tag{13}$$

we then obtain

$$\frac{d}{dt}v(t) = (A + S - qI)v(t) + (B_1 - pI)v(t) + qu(t) \quad (t>0), \quad \lim_{t\to 0^+} v(t) = v_0, \tag{14}$$

where $v_0(n) = (n+1)u_0(n+1)$ and where

$$[B_1 f]_n = p \sum_{s=0}^{n+1} sb(s) f(n+1-s) \qquad (n = 0, 1, \ldots) . \tag{15}$$

Remark 3. System (14) is a first step to derive an equation for the first moment of the neutron population $\langle n\rangle(t)$, where

$$\langle n\rangle(t) = \sum_{n=0}^{\infty} (n+1) u(n+1, t) \tag{16}$$

(see [**1**], [**2**]).

Remark 4. System (14) was derived from (5) in a heuristic way. In fact, (14) *formally* follows from (5) by applying to both sides of the two (5) the unbounded operator F, where $[Ff]_n = (n+1)f(n+1)$. The rigorous justification of system (14) is one of the aims of this paper.

It follows from definitions (15), [**1**],

Lemma 4. $B_1 \in \mathscr{B}(X)$, $B_1[X_+] \subset X_+$, $\|B_1 f\| = p\bar{\nu}\|f\|$, $\forall f \in X_+$, *where* $\bar{\nu} = \sum_{s=0}^{\infty} sb(s)$ *is the mean number of neutrons emitted if one neutron is absorbed. Moreover,*

$$A = \chi + (B_1 - pI) = (G + S - qI) + (B_1 - pI) \in \mathscr{G}(1, (\bar{\nu}-1)/l) , \tag{17}$$

$$\|\exp[tA] f\| = \exp\left[\frac{\bar{\nu}-1}{l}\, t\right] \|f\| \qquad \forall f \in X_+ , \tag{18}$$

where the semigroup $\exp[tA]$ *maps* X_+ *into itself.* ∎

We now consider the following initial-value problem

$$\frac{\mathrm{d}}{\mathrm{d}t} v(t) = Av(t) + q\, u(t) \quad (t > 0) , \qquad \lim_{t\to 0_+} v(t) = v_0 , \tag{19}$$

where $u(t)$ is given by (11). Due to (17), the solution of (19) has the form

$$v(t) = \exp[tA] v_0 + q \int_0^t \exp[(t-t')A] u(t')\, \mathrm{d}t' , \tag{20}$$

provided that $v_0 \in D(A) = D(\chi) = D(G)$ ([**12**]$_2$, p. 486).

Under the assumptions of Lemma 3 (see relations (12)) and if $v_0 \in D_+(G)$, (20) shows that $v(t) \in D_+(G)$ at any $t \geqslant 0$. Moreover, we obtain from (20)

$$\|v(t)\| = \exp\left[\frac{\bar{\nu}-1}{l}t\right]\|v_0\| + q\int_0^t \exp\left[\frac{\bar{\nu}-1}{l}(t-t')\right]dt'$$

because of (18). Hence,

$$\frac{d}{dt}\|v(t)\| = \frac{\bar{\nu}-1}{l}\|v(t)\| + q \qquad (t>0), \qquad \lim_{t\to 0_+}\|v(t)\| = \|v_0\| . \tag{21}$$

The first of (21) has the standard form of the nuclear reactor kinetics equation [**2**].

We may summarize the preceding results as follows.

Theorem 1. *If $u_0 \in D_+(G)$ with $\|u_0\| = 1$ and if $v_0 \in D_+(G)$, we then have*

(a) *the solution $u(t)$ of (10) is such that $u(t) \in D_+(G)$, $\|u(t)\| = 1$ and $Au(t) = Gu(t)$ at any $t \geqslant 0$;*

(b) *the solution $v(t)$ of (19) is such that $v(t) \in D_+(G)$ and $Av(t) = Gv(t)$ at any $t \geqslant 0$ (hence, $v(t)$ also satisfies (14));*

(c) $\|v(t)\| = \sum_{n=0}^{\infty} v(n, t)$ *satisfies system (21).* ∎

Remark 5. Theorem 1 does *not* imply that relation (13) is true and that, consequently, $\|v(t)\| = \langle n \rangle(t)$. Hence, we can *not* infer that $\langle n \rangle(t)$ satisfies system (21). Relation (13) must be proved « a posteriori » (see Sect. 4 and 5).

4. - The « approximate » solutions $u_r(t)$ and $v_r(t)$.

Let us consider the following initial-value problems

$$\frac{d}{dt}u_r(t) = \chi_r u_r(t) \qquad (t>0), \qquad \lim_{t\to 0_+} u_r(t) = u_0 , \tag{22}$$

$$\frac{d}{dt}v_r(t) = A_r v_r(t) + q\, u_r(t) \qquad (t>0), \qquad \lim_{t\to 0^+} v_r(t) = v_0 , \tag{23}$$

where we assume that both u_0 and v_0 belong to $D = D(H) = D(G_r)$ and where

(24) $$\chi_r = G_r + S - qI\,, \qquad \Lambda_r = \chi_r + B_1 - pI\,, \qquad D(\chi_r) = D(\Lambda_r) = D\,.$$

Remark 6. As it will be proved in the sequel, u_r and v_r «approximate» u and v if r is close to 1 (compare (22) with (10) and (23) with (20)).

We have

Lemma 5. *$\chi_r \in \mathcal{G}(1, 0)$, $\Lambda_r \in \mathcal{G}(1, (\bar{\nu}-1)/l)$ and both the semigroups* $\exp[t\chi_r]$ *and* $\exp[t\Lambda_r]$ *map X_+ into itself*. ■

In fact, $G_r \in \mathcal{G}(1, 0)$ and $\|S\| = q$. Hence ([**12**]$_2$, p. 495), $G_r + S \in \mathcal{G}(1, q)$. On the other hand

(25) $$\exp[t\chi_r] = \exp[-qt]\exp[t(G_r + S)]\,,$$

since qI commutes with $(G_r + S)$. We conclude that $\chi_r \in \mathcal{G}(1, 0)$. Moreover, since $\exp[tG_r][X_+] \subset X_+$ (see Lemma 1) and $S[X_+] \subset X_+$ (see Lemma 2), relation (25) shows that $\exp[t\chi_r]$ maps X_+ into itself ([**12**]$_2$, p. 495). The operator Λ_r can be dealt with in an anologous way. ■

The following theorem is a direct consequence of Lemma 5.

Theorem 2. *If u_0 and v_0 both belong to D_+, then*

(26) $$u_r(t) = \exp[t\chi_r]u_0 \in D_+ \qquad \forall t \geq 0\,,$$

(27) $$v_r(t) = \exp[t\Lambda_r]v_0 + q\int_0^t \exp[(t-t')\Lambda_r]u_r(t')\,dt' \in D_+\,, \qquad t \geq 0\,,$$

for any $r \in [0, 1)$. ■

The importance of $u_r(t)$ and of $v_r(t)$ is due to the fact that $u_r(t)$ and $v_r(t)$ « approximate » $u(t)$ and $v(t)$ in the following sense.

Theorem 3. *If u_0 and v_0 belong to D_+, then*

(28) $$\lim_{r\to 1_-} u_r(t) = u(t)\,, \qquad \lim_{r\to 1_-} v_r(t) = v(t)\,,$$

uniformly with respect to t in each finite interval $[0, \bar{t}]$. ■

Theorem 3 is a direct consequence of the following Lemma 6 with $C = S - qI$ and with $C = S - qI + B_1 - pI$.

Lemma 6. *If* $C \in \mathscr{B}(X)$ ([**12**]$_2$, p. 149), *then*

$$(29) \qquad \lim_{r \to 1_-} \exp[t(G_r + C)]f = \exp[t(G + C)]f, \qquad \forall f \in X,$$

uniformly with respect to t in each finite interval $[0, \bar{t}]$. ■

Lemma 6 follows from (7) and from Theorem 2.16 of [**12**]$_2$ (p. 502), (see also [**13**], Theorem 2). ■

Remark 7. The approximate solution $u_r(t)$ can be profitably used to derive specific properties of the exact solution $u(t)$. This is due to the fact that $u_r(t) \in D_+ = D \cap X_+$ at any $t \geq 0$ (see (26)) where the structure of D is completely known. On the other hand, $u(t) \in D_+(G) = D(G) \cap X_+$ at any $t \geq 0$ (see (12)) and we only know that $D(G)$ satisfies the relation $D \subset D(G) \subset D(A)$.

5. - $u(t)$, $v(t)$ and $\langle n \rangle (t)$.

We shall now exploit Theorem 3 to show that $u(t)$ and $v(t)$ (see Theorem 1) satisfy the relation

$$(30) \qquad Jv(t) = Yu(t), \qquad \forall t \geq 0,$$

where

$$(31) \quad [Jf]_n = f(n)/(n+1), \; [Yf]_n = f(n+1) \quad (n = 0, 1, ...), \; D(J) = D(Y) = X,$$

and where it follows directly from definition (31) that

$$(32) \qquad J \in \mathscr{B}(X), \qquad Y \in \mathscr{B}(X), \qquad \|J\| \leq 1, \qquad \|Y\| = 1,$$

$$(33) \qquad J[D] \subset D, \qquad Y[D] \subset D.$$

Remark 8. Relation (30) is equivalent to (13). Hence, if (30) is true, (16) shows that $\|v(t)\| = \langle n \rangle (t)$. Consequently, $\langle n \rangle (t)$ satisfies system (21) and the nuclear reactor kinetics equation is a rigorous consequence of the Chapman-Kolmogorov system.

In order to prove (30), we introduce the following maps from $[0, +\infty]$ to X

$$(34) \qquad w(t) = Jv(t) - Yu(t), \qquad w_r(t) = Jv_r(t) - Yu_r(t).$$

Due to (32), we have from (28) and from (34)

$$\lim_{r\to 1_-} w_r(t) = w(t) \tag{35}$$

uniformly with respect to $t \in [0, \bar{t}]$, provided that u_0 and v_0 belong to D_+ (see Theorem 3).

We shall now derive an equation which gives the evolution of $w_r(t)$.

Since $J(D) \subset D$ and $Y[D] \subset D$ (see (33)), we have for any $f \in D$ and for any $g \in X$

$$\begin{cases} YHf = HYf + pYf\,, & YKf = KYf + B_0 Yf \\ JHf = HJf\,, & JKf = KJf + (B_0 - B_1)Jf - J(B_1 - B_2)Jf\,, \end{cases} \tag{36}$$

$$\begin{cases} YSg = SYg = qIg\,, & JSg = SJg - JSJg \\ JB_1 g = B_1 Jg + J(B_1 - B_2)Jg\,, \end{cases} \tag{37}$$

where

$$\begin{cases} [B_0 g]_n = p\sum_{s=0}^{n+1} b(s)\, g(n+1-s)\,, & D(B_0) = X\,,\ \|B_0\| \leqslant p \\ [B_2 g]_n = p\sum_{s=0}^{n+1} s^2 b(s)\, g(n+1-s)\,, & D(B_2) = X\,, \end{cases} \tag{38}$$

$$\|B_2\| \leqslant p[\bar{\nu}^2] = p\sum_{s=0}^{\infty} s^2 b(s)\,. \tag{39}$$

By using relations (36) and (37) and by taking into account that both Y and J belong to $\mathscr{B}(X)$, we obtain from (22), (23) and from the second of (34)

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} w_r(t) &= (\chi_r + rB_0 - pI - JS) w_r(t) + \\ &+ (1-r)[B_1 + J(B_1 - B_2)] Jv_r(t) \quad (t > 0)\,, \qquad \lim_{t\to 0_+} w_r(t) = 0\,. \end{aligned} \tag{40}$$

We note that the second of (40) follows from the assumption $Jv_0 = Yu_0$ (see (14) of Sect. **3**).

We have

Lemma 7. $(\chi_r + rB_0 - pI - JS) \in \mathscr{G}(1, q)$, $\forall r \in [0, 1)$. ∎

In fact,

$$\exp[t(\chi_r + rB_0 - pI - JS)] = \exp[-pt]\exp[t(\chi_r + rB_0 - JS)], \tag{41}$$

where $(\chi_r + rB_0 - pI - JS) \in \mathscr{G}(1, rp + q)$ since $\chi_r \in \mathscr{G}(1, 0)$ (see Lemma 5) and $r\|B_0\| \leqslant rp$, $\|JS\| \leqslant \|S\| = q$. Relation (41) then shows that

$$(\chi_r + rB_0 - JS - pI) \in \mathscr{G}(1, q - (1-r)p) \subset \mathscr{G}(1, q). \quad ∎$$

Due to Lemma 7, we have from (40)

$$w_r(t) = (1-r)\int_0^t \exp[(t-t')(\chi_r + rB_0 - pI - JS)]\{B_1 + J(B_1 - B_2)\} Jv_r(t')\,dt'$$

and also

$$\|w_r(t)\| \leqslant (1-r)\int_0^t \exp[q(t-t')]\|B_1 + J(B_1 - B_2)\|\,\|v_r(t')\|\,dt', \tag{42}$$

where, due to (27) and to Lemma 5,

$$\|v_r(t')\| \leqslant \exp\left[\frac{\bar{\nu}-1}{l}t'\right]\|v_0\| + q\int_0^{t'} \exp\left[\frac{\bar{\nu}-1}{l}(t'-t'')\right]\|u_0\|\,dt''. \tag{43}$$

Inequalities (42) and (43) imply that

$$\lim_{r\to 1_-} w_r(t) = 0 \tag{44}$$

uniformly with respect to $t \in [0, \bar{t}]$.

Relation (30) is a consequence of (35) and of (44).

We may summarize the preceding results as follows.

Main Theorem. *If* $u_0 \in D_+$, $v_0 \in D_+$, $\|u_0\| = 1$ *and if* $Jv_0 = Yu_0$, *we have at any* $t \geqslant 0$

(a) $u(t) \in D_+(G)$, $\|u(t)\| = 1$, $Au(t) = Gu(t)$, *where* $u(t)$ *is the solution of* (10);

(b) $v(t) \in D_+(G)$, $Av(t) = Gv(t)$, *where* $v(t)$ *is the solution of* (19);

(c) $Jv(t) = Yu(t)$.

Moreover, $\|v(t)\|$ *satisfies system* (21) *and*

$$\|v(t)\| = \sum_{n=0}^{\infty} (n+1)\, u(n+1, t) = \langle n \rangle(t) \,. \qquad \blacksquare$$

We conclude that the Chapman-Kolmogorov system (5) admits a solution $u(t) = \{u(n, t), n = 0, 1, \ldots\}$ such that the first moment of the neutron population (16) exists and it satisfies the nuclear reactor system

$$\frac{\mathrm{d}}{\mathrm{d}t} \langle n \rangle(t) = \frac{\bar{\nu} - 1}{l} \langle n \rangle(t) + q \qquad (t > 0)\,, \qquad \lim_{t \to 0_+} \langle n \rangle(t) = \langle n \rangle_0 = \|v_0\| \,.$$

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Summary.

We prove that the kinetic equation for the first moment of the neutron population in a multiplying assembly can be derived from the Chapman-Kolmogorov equations in a rigorous way. The proof involves a detailed study of a suitable approximate solution of the Chapman-Kolmogorov system, whose properties can be profitably used to obtain the corresponding properties of the exact solution.

Sunto.

Si prova che l'equazione, che regola l'evoluzione del momento di ordine uno della popolazione neutronica in un mezzo moltiplicante, può essere ottenuta in modo rigoroso dal sistema Chapman-Kolmogorov. La dimostrazione si fonda sullo studio di una opportuna soluzione approssimata del sistema di Chapman-Kolmogorov, le proprietà della quale possono essere sfruttate per ricavare le corrispondenti proprietà della soluzione esatta.

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