Riv. Mat. Univ. Parma 9 (1958), 113-123

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Some Integral Representations Involving Generalised Hankel-Transform. (**)

1. - Introduction.

AGARWAL [1] gave a generalisation of the well-known HANKEL-transform viz.,

$$f(x) = \int_0^\infty \sqrt{xy} \cdot J_\nu(xy) \cdot g(y)\, \mathrm{d}y \tag{1}$$

by means of the integral equation

$$f(x) = (1/2)^\lambda \int_0^\infty (xy)^{\lambda+(1/2)} \cdot J_\lambda^\mu(x^2y^2/4) \cdot g(y)\, \mathrm{d}y, \tag{2}$$

where $J_\lambda^\mu(x)$ is BESSEL-MAITLAND function defined by the series

$$J_\lambda^\mu(x) = \sum_{r=0}^\infty \frac{(-x)^r}{r!\, \Gamma(1+\lambda+\mu r)} \qquad (\mu > 0)\,. \tag{3}$$

For $\mu = 1$, **1**(2) reduces to **1**(1).

He gave the inversion formula and some theorems for this generalised HANKEL-transform.

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(**) Received November 10, 1957.

An equivalent and more convenient form of **1**(2) is

$$\psi_{\lambda,\mu,\nu} = \int_0^\infty (xy)^\nu \cdot J_\lambda^\mu(xy) \cdot g(y)\, dy\,. \tag{4}$$

Recently, I have given a number of properties for this generalised HANKEL-transform. In this paper, I have obtained some integral representations for the same and have used them (i) to evaluate some integrals involving BESSEL-MAITLAND function $J_\lambda^\mu(x)$ defined by **1**(3) and (ii) to find out the operational forms of some functions.

We shall call $\Phi(p)$ the LAPLACE-transform or simply the operational form of $f(x)$, if the relation

$$\Phi(p) = \int_0^\infty e^{-px} f(x)\, dx$$

holds and shall write this as

$$\Phi(p) \doteqdot f(x)\,.$$

2. – First Representation.

Let us consider the integral (AGARWAL [2])

$$\int_a^\infty J_{\lambda+\mu}^\mu(xy)\, dx = J_\lambda^\mu(ay)/y,$$

valid for $0 < \mu \leqslant 1$ with an additional condition $\Re(\lambda) > -3/2$ in case $\mu = 1$. Multiplying both sides by $y^\nu\, g(y)$ and integrating from 0 to ∞, we have

$$\int_a^\infty \psi_{\lambda+\mu,\mu,\nu}(x) \cdot x^{-\nu}\, dx = a^{1-\nu} \cdot \psi_{\lambda,\mu,\nu-1}(a), \tag{1}$$

provided (i) the above integral exists, (ii) the integral

$$\int_0^\infty (xy)^\nu \cdot J_{\lambda+\mu}^\mu(xy) \cdot g(y)\, dy$$

is absolutely convergent and (iii) $0 < \mu \leqslant 1$ with an additional condition $\Re(\lambda) > -3/2$ in case $\mu = 1$.

Applications: We shall now use this result to evaluate some integral s

Example 1. Let $g(y)$ be a function defined by

$$g(y) = \begin{cases} y^{(\lambda+1)/\mu-1}(1-y^{1/\mu})^{c-1} & \text{when } 0 \leqslant y \leqslant 1, \\ 0 & \text{when } y > 1. \end{cases}$$

Therefore (KUMAR [2], 3(7)):

$$\psi_{\lambda+n\mu-m,\mu,n}(x) = \int_0^1 (xy)^n \cdot J^{\mu}_{\lambda+n\mu-m}(xy) \cdot y^{(\lambda+1)/\mu-1}(1-y^{1/\mu})^{c-1}\, \mathrm{d}y =$$

$$= \mu^{m+1} \cdot \Gamma(c) \cdot x^{-(\lambda-m)/\mu} \cdot (x^{1-1/\mu}\, \mathrm{d}/\mathrm{d}x)^m \left[x^{\lambda/\mu+n} J^{\mu}_{\lambda+c+n\mu}(x)\right]$$

and

$$\psi_{\lambda+n\mu-m+\mu,\mu,n+1}(x) = \mu^{m+1} \cdot \Gamma(c) \cdot x^{-(\lambda-m)/\mu}(x^{1-1/\mu}\, \mathrm{d}/\mathrm{d}x)^m \left[x^{\lambda/\mu+n+1} J^{\mu}_{\lambda+c+n\mu+\mu}(x)\right],$$

provided $\Re(\lambda + n\mu) > -1$, $\Re(c) > 0$, $\mu > 0$, and m, n are any positive integers (including zero). Therefore, **2** (1),

$$\int_a^\infty x^{-(\lambda-m)/\mu-n-1}(x^{1-1/\mu} \cdot \mathrm{d}/\mathrm{d}x)^m \left[x^{\lambda/\mu+n+1}\, J^{\mu}_{\lambda+c+n\mu+\mu}(x)\right] \mathrm{d}x = \tag{2}$$

$$= a^{-(\lambda-m)/\mu-n} \left[(a^{1-1/\mu} \cdot \mathrm{d}/\mathrm{d}a)^m \left\{ a^{\lambda/\mu+n}\, J^{\mu}_{\lambda+c+n\mu}(a) \right\}\right],$$

provided $\Re(\lambda + n\mu) > -1$, $\Re(c) > 0$, $0 < \mu < 1$, and m, n are any positive integers (including zero) .

Example 2. Let us take

$$g(y) = y^{(\lambda+1)/\mu-1}\, e^{-y^{1/\mu}},$$

then (KUMAR [5], 3(10))

$$\psi_{\lambda+n\mu-m,\mu,n}(x) = \int_0^\infty (xy)^n \cdot J^{\mu}_{\lambda+n\mu-m}(xy) \cdot y^{(\lambda+1)/\mu-1} \cdot e^{-y^{1/\mu}}\, \mathrm{d}y =$$

$$= \mu^{m+1} \cdot x^{-(\lambda-m)/\mu}(x^{1-1/\mu} \cdot \mathrm{d}/\mathrm{d}x)^m \left[x^{\lambda/\mu+n} \cdot e^{-x}\right]$$

and, hence,

$$\psi_{\lambda+n\mu-m+\mu,\mu,n+1}(x) = \mu^{m+1}\cdot x^{-(\lambda-m)/\mu}(x^{1-1/\mu}\cdot \mathrm{d}/\mathrm{d}x)^m\,[x^{\lambda/\mu+n+1}\cdot e^{-x}],$$

provided $\Re(\lambda + n\mu) > -1$, and m, n are any positive integers (including zero). Therefore, **2**(1),

$$\text{(3)}\qquad \int_a^\infty x^{-(\lambda-m)/\mu-n-1}\cdot(x^{1-1/\mu}\cdot \mathrm{d}/\mathrm{d}x)^m\,[x^{\lambda/\mu+n+1}\cdot e^{-x}]\,\mathrm{d}x = $$
$$= a^{-(\lambda-m)/\mu-n}\cdot(a^{1-1/\mu}\cdot \mathrm{d}/\mathrm{d}a)^m\{\,a^{\lambda/\mu+n}e^{-a}\,\},$$

provided $\Re(\lambda + n\mu) > -1$, $0 < \mu < 1$, and m, n are positive integers (including zero).

3. – Second Representation.

Let us consider the integral

$$\int_0^1 t^\lambda\,(1-t)^{c-1}\cdot J_\lambda^\mu\left((zt)^\mu\right)\,\mathrm{d}t = \Gamma(c)\cdot J_{\lambda+c}^\mu(z^\mu),$$

provided $\Re(\lambda) > -1$ and $\Re(c) > 0$, obtained by writing the BESSEL-MAITLAND function in the form **1**(3) and integrating term by term, a process easily justifiable.

This can be written in the form

$$\int_0^1 t^{(\lambda+1)/\mu-1}\cdot(1-t^{1/\mu})^{c-1}\cdot J_\lambda^\mu(xyt)\,\mathrm{d}t = \mu\cdot\Gamma(c)\cdot J_{\lambda+c}^\mu(xy),$$

valid for $\Re(\lambda) > -1$ and $\Re(c) > 0$.

Multiplying both sides by $(xy)^\nu\cdot g(y)$ and integrating from 0 to ∞, we get

$$\text{(1)}\qquad \int_0^1 t^{(\lambda+1)/\mu-\nu-1}\cdot(1-t^{1/\mu})^{c-1}\cdot\psi_{\lambda,\mu,\nu}(xt)\,\mathrm{d}t = \mu\cdot\Gamma(c)\cdot\psi_{\lambda+c,\mu,\nu}(x),$$

provided (i) the above integral exists, (ii) the integral

$$\int_0^\infty (xyt)^\nu\cdot J_\lambda^\mu(xyt)\cdot g(y)\,\mathrm{d}y$$

is absolutely convergent, and (iii) $\Re(\lambda) > -1$, $\Re(c) > 0$.

This can also be written in the form

$$(2)\quad \int_0^{\pi/2} (\sin\theta)^{2\lambda-2\mu\nu+1}\cdot(\cos\theta)^{2c-1}\cdot\psi_{\lambda,\mu,\nu}(x\cdot\sin^{2\mu}\theta)\,d\theta = (1/2)\cdot\Gamma(c)\cdot\psi_{\lambda+c,\mu,\nu}(x),$$

valid under the conditions stated above.

Applications: We shall now use this result to evaluate some integrals.

Example 1. Let

$$g(y) = \begin{cases} y^{(\lambda+1)/\mu+n-\nu-1}(1-y^{1/\mu})^{c-1} & \text{when } 0 \leqslant y \leqslant 1, \\ 0 & \text{when } y > 1; \end{cases}$$

then (KUMAR [5], 3(7)):

$$\psi_{\lambda+n\mu-c,\mu,\nu}(x) = \int_0^1 (xy)^\nu\cdot J^\mu_{\lambda+n\mu-c}(xy)\cdot y^{(\lambda+1)/\mu+n-\nu-1}\cdot(1-y^{1/\mu})^{c-1}\,dy =$$

$$= \mu^{c+1}\cdot\Gamma(c)\cdot x^{-(\lambda-c)/\mu-n+\nu}\cdot(x^{1-1/\mu}\cdot d/dx)^c\,[x^{\lambda/\mu+n}\cdot J^\mu_{\lambda+c+n\mu}(x)],$$

provided $\Re(\lambda+n\mu) > -1$, $\mu > 0$, and n, c are any positive integers (n can be equal to zero but $c \neq 0$), and

$$\psi_{\lambda+n\mu,\mu,\nu}(x) = \int_0^1 (xy)^\nu\cdot J^\mu_{\lambda+n\mu}(xy)\cdot y^{(\lambda+1)/\mu+n-\nu-1}\cdot(1-y^{1/\mu})^{c-1}\,dy =$$

$$= \mu\cdot\Gamma(c)\cdot x^\nu\cdot J^\mu_{\lambda+c+n\mu}(x),$$

provided $\Re(\lambda+n\mu) > -1$, $\Re(c) > 0$ and n is any positive integer (including zero). Therefore, by **3**(1),

$$(3)\quad \int_0^1 t^{(\lambda-c+1)/\mu+n-1}\cdot(1-t^{1/\mu})^{c-1}(x^{1-1/\mu}\cdot d/dx)^c\,[x^{\lambda/\mu+n}\cdot J^\mu_{\lambda+c+n\mu}(xt)\,dt =$$

$$= \Gamma(c)\cdot\mu^{1-c}\cdot x^{(\lambda-c)/\mu+n}\cdot J^\mu_{\lambda+c+n\mu}(x),$$

provided $\Re(\lambda+n\mu-c) > -1$, $\mu > 0$, and n, c being any positive integers (n can be equal to zero but $c \neq 0$).

Example 2. Let us consider the integral (KUMAR [5], 3(3)):

$$\int_0^\infty (xy)^{n+m/2} \cdot J^2_{2n+m-c+1}(xy) \cdot e^{-y} \cdot He_m(\sqrt{2y})\, dy =$$

$$= \sqrt{\pi} \cdot 2^{c-n-(m+1)/2} \cdot x^{(c-1)/2} \cdot (\sqrt{x} \cdot d/dx)^c \left[x^{(n+m)/2+1/4} \cdot J_{n+1/2}(\sqrt{x})\right],$$

where m, n and c are any positive integers (including zero), so that we take this as $\psi_{2n+m-c+1,2,n+m/2}(x)$ we have (KUMAR [1], 4(6)):

$$\psi_{2n+m+1,2,n+m/2}(x) = \int_0^\infty (xy)^{n+m/2} \cdot J^2_{2n+m+1}(xy) \cdot e^{-y} \cdot He_m(\sqrt{2y})\, dy =$$

$$= \sqrt{\pi} \cdot (1/2)^{n+(m+1)/2} \cdot x^{(n+m)/2-1/4} \cdot J_{n+1/2}(\sqrt{x}),$$

n and m being any positive integers (including zero). Therefore, by **3**(1),

$$(4) \qquad \int_0^1 t^{(n+m-c)/2-1/4} \cdot (1-\sqrt{t})^{c-1} \cdot (\sqrt{x} \cdot d/dx)^c \left[x^{(n+m)/2+1/4} \cdot J_{n+1/2}(\sqrt{xt})\right] dt =$$

$$= 2^{1-c} \cdot \Gamma(c) \cdot x^{(n+m-c)/2+1/4} \cdot J_{n+1/2}(\sqrt{x}),$$

provided $n + (m-c)/2 > -1$, and m, n, c are any positive integers (m, n can be equal to zero but $c \neq 0$).

Example 3. Let us consider the integral (KUMAR [5], 3(9))

$$\int_0^\infty (xy)^{n-1/2} \cdot J^3_{3n-c+1/2}(xy) \cdot e^{-y}\, dy =$$

$$= 2\pi \cdot 3^{c-n-1} \cdot x^{(c-2)/3} \cdot (x^{2/3} \cdot d/dx)^c \left[x^{n/3+1/6} \cdot J_{1/6+n,-1/6+n}(x^{1/3})\right],$$

where n and c are any positive integers (including zero). If we take this as $\psi_{3n-c+1/2,3,n-1/2}(x)$, we have (KUMAR [4], 4(8)):

$$\psi_{3n+1/2,3,n+1/2}(x) = \int_0^\infty (xy)^{n-1/2} \cdot J^3_{3n+1/2}(xy) \cdot e^{-y}\, dy =$$

$$= 3^{-n-1} \cdot 2\pi \cdot x^{n/3-1/2} \cdot J_{n+1/6,n-1/6}(x^{1/3}),$$

n being any positive integer (including zero). Therefore, by **3**(1),

$$(5) \qquad \int_0^1 t^{(n-c)/3-1/2}\cdot(1-t^{1/3})^{c-1}\cdot(x^{2/3}\cdot \mathrm{d}/\mathrm{d}x)^c\left\{x^{n/3+1/6}\cdot J_{n+1/6,\,n-1/6}\left((xt)^{1/3}\right)\right\}\mathrm{d}t =$$

$$= 3^{1-c}\Gamma(c)\cdot x^{(n-c)/3+1/6}\cdot J_{n+1/6,\,n-1/6}(x^{1/3}),$$

where n and c are any positive integers (n can be equal to zero but $c \neq 0$) and $3n - c > -3/2$.

Example 4. Let us consider the integral (KUMAR [5], 3(10))

$$\int_0^\infty (xy)^{(\lambda+1)/\mu+n-1}\cdot J^{\mu}_{\lambda+n\mu-c}(xy)\cdot e^{-y^{1/\mu}}\,\mathrm{d}y =$$

$$= \mu^{c+1}\cdot x^{(c+1)/\mu-1}\cdot(x^{1-1/\mu}\cdot\mathrm{d}/\mathrm{d}x)^c\left[x^{\lambda/\mu+n}\cdot e^{-x}\right],$$

provided n and c are any positive integers (including zero) and $\Re(\lambda+n\mu) > -1$. If we take this as $\psi_{\lambda+n\mu-c,\mu,(\lambda+1)/\mu+n-1}(x)$, we get

$$\psi_{\lambda+n\mu,\,\mu,(\lambda+1)/\mu+n-1}(x) = \int_0^\infty (xy)^{(\lambda+1)/\mu+n-1}\cdot J^{\mu}_{\lambda+n\mu}(xy)\cdot e^{-y^{1/\mu}}\,\mathrm{d}y = \mu\cdot x^{(\lambda+1)/\mu+n-1}\cdot e^{-x},$$

n being any positive integers (including zero) and $\Re(\lambda+n\mu) > -1$. Therefore, **3**(1),

$$(6) \qquad \int_0^1 t^{(\lambda-c+1)/\mu+n-1}\cdot(1-t^{1/\mu})^{c-1}\cdot(x^{1-1/\mu}\cdot\mathrm{d}/\mathrm{d}x)^c\left[x^{\lambda/\mu+n}\cdot e^{-xt}\right]\mathrm{d}t =$$

$$= \mu^{1-c}\cdot\Gamma(c)\cdot x^{(\lambda-c)/\mu+n}\cdot e^{-x},$$

provided n and c are any positive integers (n can be equal to zero but $c \neq 0$) and $\Re(\lambda+n\mu-c) > -1$.

4. – Third Representation.

It is easy to show by term by term integration that

$$\int_0^\infty x^{\lambda}\cdot J^{\mu}_{\lambda}(x^{\mu}\,y)e^{-px}\,\mathrm{d}x = p^{-\lambda-1}\,e^{-yp^{-\mu}}$$

valid for $\Re(\lambda) > -1$.

Multiplying both sides by $y^{\nu}\cdot g(y)$ and integrating from 0 to ∞, we get

$$\int_0^\infty x^{\lambda-\mu\nu}\cdot e^{-px}\cdot \psi_{\lambda,\mu,\nu}(x^{\mu})\,dx = p^{-\lambda-1}\int_0^\infty e^{-yp^{-\mu}}\cdot y^{\nu}\cdot g(y)\,dy,$$

provided the integrals converge and the change in the order of integration is permissible.

Hence, we have

$$(1)\qquad \int_0^\infty x^{\lambda-\mu\nu}\cdot e^{-px}\cdot \psi_{\lambda,\mu,\nu}(x^{\mu})\,dx = p^{-\lambda-1}\cdot G_{\nu}(p^{-\mu}),$$

where $G_{\nu}(p)$ is defined as

$$G_{\nu}(p) = \int_0^\infty e^{-yp}\cdot y^{\nu}\cdot g(y)\,dy,$$

provided: (i) the above integral exists, (ii) the integral

$$\int_0^\infty (x^{\mu}\cdot y)^{\nu}\cdot J_{\lambda}^{\mu}(x^{\mu}\cdot y)\cdot g(y)\,dy$$

is absolutely convergent, and (iii) $\Re(\lambda) > -1$.

Applications. This result can be conveniently used to determine the operational forms of some functions for knowing the operational form of $y^{\nu}\cdot g(y)$, the operational form of $x^{\lambda-\mu\nu}\cdot\psi_{\lambda,\mu,\nu}(x^{\mu})$ can easily be written down.

Example 1. Let us take

$$g(y) = y^{\lambda-\nu-1}\cdot e^{-y\sqrt{2}}\cdot \sin(y\sqrt{2}),$$

then (KUMAR [3], 5(7)):

$$\psi_{(\lambda-1)/2,1/2,\nu}(x) = (1/\sqrt{2})\,(-1)^{\lambda-1}\cdot x^{\nu}\cdot(d^{\lambda-1}/dx^{\lambda-1})\,[\cos(x^2/4)\{1/2 - C(x/\sqrt{2\pi})\} + $$
$$+ \sin(x^2/4)\{1/2 - S(x/\sqrt{2\pi})\}],$$

provided $\lambda=1, 2, 3, \ldots$, $C(x)$ and $S(x)$ being FRESNEL's cosine and sine integrals (MAGNUS [7], p. 96). Therefore, by 4(1) the image of

$$2^{\lambda-3/2}\cdot(-1)^{\lambda-1}\cdot x^{(\lambda-1)/2}\cdot(x^{1/2}\,d/dx)^{\lambda-1}\,[\cos(x/4)\{1/2 - C(\sqrt{x/(2\pi)})\} + $$
$$+ \sin(x/4)\{1/2 - S(\sqrt{x/(2\pi)})\}]$$

is

$$p^{-(\lambda+1)/2}\int_0^\infty e^{-yp^{-1/2}}\cdot y^{\lambda-1}\cdot e^{-y\sqrt{2}}\cdot\sin(y\sqrt{2})\,dy,$$

i.e.

$$\Gamma(\lambda)\cdot[\{(p+\sqrt{2})^{-1/2}+i\sqrt{2}\}^{\lambda}-\{(p+\sqrt{2})^{-1/2}-i\sqrt{2}\}^{\lambda}]/ \\ /[2ip^{(\lambda+1)/2}\cdot\{(p+\sqrt{2})^{-1/2}+2\}^{\lambda}]$$

(MAGNUS [7], p. 125), provided $\lambda=1, 2, 3, \ldots$. In particular, when $\lambda=1$, we get the image of

(3) $$(1/\sqrt{2})\cdot[\cos(x/4)\{1/2-C(\sqrt{x/(2\pi)})\}+\sin(x/4)\{1/2-S(\sqrt{x/(2\pi)})\}]$$

as

(4) $$\sqrt{2}/[p\{(p+\sqrt{2})^{-1}+2\}].$$

Example 2. Let us take

$$g(y)=y^{\lambda-\nu-1}\cdot e^{-y\sqrt{2}}\cdot\cos(y\sqrt{2}),$$

then, by (KUMAR [6], 5(5))

$$\psi_{(\lambda-1)/2,1/2,\nu}(x)=(1/\sqrt{2})(-1)^{\lambda-1}\cdot x^{\nu}(d^{\lambda-1}/dx^{\lambda-1})[\cos(x^2/4)\{1/2-S(x/\sqrt{2\pi})- \\ -\sin(x^2/4)\{1/2-C(x/\sqrt{2\pi})\}],$$

provided $\lambda=1, 2, 3, \ldots$, $C(x)$ and $S(x)$ being FRESNEL's cosine and sine integrals (MAGNUS [7], p. 96). Therefore, by **4**(1) the image of

$$2^{\lambda-3/2}\cdot(-1)^{\lambda-1}\cdot x^{(\lambda-1)/2}\cdot(\sqrt{x}\cdot d/dx)^{\lambda-1}[\cos(x/4)\{1/2-S(\sqrt{x/(2\pi)})\}- \\ -\sin(x/4)\{1/2-C(\sqrt{x/(2\pi)})\}]$$

is

$$p^{-(\lambda+1)/2}\cdot\int_0^\infty e^{-yp-1/2}\cdot y^{\lambda-1}\cdot e^{-y\sqrt{2}}\cdot\cos(y\sqrt{2})\,dy$$

i.e.

$$(5)\qquad \Gamma(\lambda)\cdot p^{-(\lambda+1)/2}\cdot[\{(p+\sqrt{2})^{-1/2}+i\sqrt{2}\}^\lambda+$$

$$+\{(p+\sqrt{2})^{-1/2}-i\sqrt{2}\}^\lambda]/[2\{(p+\sqrt{2})^{-1}+2\}^\lambda]$$

(Magnus [7], p. 125), provided $\lambda=1, 2, 3, \ldots$. In particular, when $\lambda=1$, we get the image of

$$(1/\sqrt{2})\,[\cos(x/4)\{1/2-S(\sqrt{x/(2\pi)})\}-\sin(x/4)\{1/2-C(\sqrt{x/(2\pi)})\}]$$

as

$$(p+\sqrt{2})^{-1/2}/[p\{(p+\sqrt{2})^{-1}+2\}].$$

Example 3. Let us take

$$g(y)=y^{\lambda+\nu}\cdot K_\nu(2y),$$

then (Gupta [3])

$$\psi_{(\lambda-1)/2,1/2,0}(x)=(\sqrt{2})^{-\lambda-2\nu-3}\cdot\Gamma(\lambda+2\nu+1)\cdot e^{x^2/8}\cdot D_{-(\lambda+2\nu+1)}(x/\sqrt{2}),$$

provided $\Re(\lambda+2\nu)>-1$ and $\Re(\lambda)>-1$. Therefore, by 4(1), the image of

$$(\sqrt{2})^{-\lambda-2\nu-3}\cdot\Gamma(\lambda+2\nu+1)\cdot x^{(\lambda-1)/2}\cdot e^{x/8}\cdot D_{-(\lambda+2\nu+1)}(\sqrt{x/2})$$

is

$$p^{-(\lambda+1)/2}\cdot\int_0^\infty e^{-yp-1/2}\cdot y^{\lambda+\nu}\cdot K_\nu(2y)\,dy$$

i.e.

$$(7) \qquad (\pi/\sin \nu\pi)\cdot p^{-(\lambda+1/2}\cdot 2^{\lambda+\nu-1}\cdot (p^{3/2}\cdot \mathrm{d}/\mathrm{d}p)^{\lambda+\nu}\left[\left\{1-4^{\nu}/(p^{-1/2}+\sqrt{1/p-4})^{2\nu}\right\}\cdot \cdot\left\{1/(2\sqrt{p})-\sqrt{1/(4p)-1}\right\}^{\nu}/\sqrt{1/p-4}\right]$$

(MAGNUS [7], p. 125), provided $\Re(\lambda+2\nu)>-1$, $\Re(\lambda)>-1$ and $\lambda+\nu$ is any positive integer including zero.

My best thanks are due to Dr. R. S. VARMA for his helpful suggestions in the preparation of this paper.

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